ATLANTA REGIONAL MEETING—OCTOBER 31–NOVEMBER 1

JOURNAL
of the
AMERICAN CONCRETE INSTITUTE

(ACI PROCEEDINGS Vol. 51)

Vol. 26 April 1955 No. 8

CONTENTS

Papers and Reports....733-812

Looking Ahead....CHARLES H. SCHOLER 733

Theory of the Secondary Arch....B. F. JAKOBSEN 741

Prefabricated Factory Construction in Denmark....R. A. LARSEN and VAGN USSING 757

Strength Variations in Ready-Mixed Concrete....A. E. CUMMINGS 765

Balanced Design of Prestressed Concrete Beams....HENRY J. COWAN 773

An Interpretation of Some Published Researches on the Alkali-Aggregate Reaction. Part 2—A Hypothesis Concerning Safe and Unsafe Reactions with Reactive Silica in Concrete....T. C. POWERS and H. H. STEINOUR 785

Letters from Readers....813-820

Concrete's Etyomological Offspring.... 813

Radioactive Tracer Techniques.... 818

Current Reviews....821-832

News Letter.... 1-66

to provide a comradeship in finding the best ways to do concrete work of all kinds and in spreading that knowledge

ADDRESS: 18263 W. McNICHOLS RD., DETROIT 19, MICH.

$1.50 per copy
Extra copies to ACI Members $1.00

PREFABRICATION CUTS SCHOOL COSTS

Precast, Prestressed 'Incor' Concrete Roof Speeds Completion of Fire-Safe School, At Big Saving in Cost

● With enough children being born every fifteen minutes to fill a classroom, and a million pupils in sub-standard quarters, the nation faces a race against time in solving the school problem.

Fort Lauderdale comes up with a sound solution. Dillard Elementary School, with 21 classrooms and about 25,000 sq. ft. of floor space, was completed 18 weeks ahead of schedule. Cost, $273,000. — $38,500. under the appropriation.

Built of fire-safe concrete, speed and economy pivoted on the roof system, consisting of 32,000 sq. ft. of double-tee, precast, prestressed roof slabs, 221 in all. Each slab is 34½ ft. x 4 ft., with 23 ft. clear span and 6 ft. cantilever on one side, 4 ft. on the other.

With ten hours' steam-curing, dependable 'Incor'* high early strength produced 4000 psi in 20 hours for pretensioning and stripping immediately thereafter .... 11,000 sq. ft. of roof slabs placed in an eight-hour day.

Double-tee design gives pleasing beam effect. Quality concrete, with smooth exposed roof and ceiling surfaces, only required painting to finish.

Another example of attractive, fire-safe construction, faster and at less cost, thanks to the two I's — Ingenious design and 'Incor' performance.

*Reg. U.S. Pat. Off.

LONE STAR CEMENT CORPORATION

Offices: ABILENE, TEX. · ALBANY, N.Y. · BETHLEHEM, PA. · BIRMINGHAM · BOSTON · CHICAGO · DALLAS · HOUSTON
INDIANAPOLIS · KANSAS CITY, MO. · NEW ORLEANS · NEW YORK · NORFOLK · RICHMOND · WASHINGTON, D.C.

LONE STAR CEMENT, WITH ITS SUBSIDIARIES, IS ONE OF THE WORLD'S LARGEST CEMENT PRODUCERS: 18 MODERN MILLS, 141,600,000 SACKS ANNUAL CAPACITY

Vol. 26—No. 8 April 1955 (Proceedings Vol. 51)

JOURNAL
of the
AMERICAN CONCRETE INSTITUTE

Published by the American Concrete Institute under the authority of its Board of Direction

ACI history and objectives. The Institute was founded 1905, incorporated in the District of Columbia in 1906 as The National Association of Cement Users, the name changed 1913 by charter amendment. It was re-incorporated August 1, 1945 with these objects: "to organize the efforts of its members for a nonprofit public service in gathering, correlating, and disseminating information for the improvement of the design, construction, manufacture, use, and maintenance of concrete products and structures."

Papers, committee reports, and other contributions to the **Proceedings** pages of this Journal, as submitted in furtherance of Institute objectives, are, if published, accepted on the sole responsibility of their authors. Institute authority attaches only to standards adopted as provided in the Bylaws.

Advertising. See statement under "Alphabetical List of Advertisers" final page **News-Letter** section.

Membership dues and Journal subscription rates. Members receive the Journal as one of the considerations of dues payments. Membership grades are: Individual (U. S., its possessions, and North and Central America), $15; Individual (all other foreign countries), $12; Corporation, $50; Contributing, $100; Junior, $7.50; Student, $5. Subscription prices are $12 per year, payable in advance ($15 per year outside North America and U.S. possessions). The Journal is issued monthly, January to June and September to December (the volume year of September to June being completed by title page, contents, indexes, and closing discussion in Part 2 of the subsequent December issue).

Bound volumes 1 to 50 of PROCEEDINGS of the AMERICAN CONCRETE INSTITUTE (1905 to 1954) are for sale as far as available at prices to be had on inquiry of the Secretary-Treasurer. Special prices apply for members ordering bound volumes in addition to the monthly Journal.

Address. American Concrete Institute, 18263 W. McNichols Road, Detroit 19, Michigan. **Copyright, 1955,** American Concrete Institute. Printed in U. S. A. Entered in the Post Office at Detroit, Michigan, as mail of the second class under provisions of the Act of March 3, 1879.

President
CHARLES S. WHITNEY

Vice-Presidents
FRANK KEREKES
WALTER H. PRICE

Secretary-Treasurer
WILLIAM A. MAPLES

Technical Director
PAUL F. RICE

Directors
F. BARONA DE LA O
ARTHUR P. CLARK
HARRY C. DELZELL
PHIL M. FERGUSON
JOSEPH F. JELLEY
JAY E. JELLICK
VERNON P. JENSEN
THOMAS B. KENNEDY
WALTER J. McCOY
DOUGLAS McHENRY
H. C. ROSS
GEORGE W. WASHA

Past Presidents
A. T. GOLDBECK
HENRY L. KENNEDY
C. H. SCHOLER

Advertising Manager
ANDREW M. SMITH

Manager, Publications Sales
ISABELLE DAVIS

Membership Secretary
EVELYN SCHLITZ

American Concrete Institute

This is to Certify that

John Smith

is a

Member

of this institute and fully entitled to the privileges granted by its Charter and Bylaws

_____
President

_____
Secretary-Treasurer

Date of membership: July 1, 1952

Above is shown the membership certificate now available to all interested members. (Student members become eligible for certificates when they advance to one of the higher membership grades.) The actual certificate is 10 x 12 in. on parchment paper suitable for framing.

Return this order form together with your remittance—to avoid the added expense of billing—and your certificate will be mailed to you. Please allow 60 days for preparation of the certificate.

Please accept my order for a membership certificate at the price of $1.50.

☐ Check ☐ M.O. Enclosed

My name is to appear exactly as follows:

_____
(Print or type)

Signature.....

the ACI JOURNAL

is edited by the Secretary of the Technical Activities Committee under the direction of the Committee

BRYANT MATHER
Chairman

CHARLES S. WHITNEY
(ex officio)

WILLIAM A. MAPLES
Secretary

PHIL M. FERGUSON

JAMES A. McCARTHY

W. J. McCOY

DOUGLAS McHENRY

L. H. TUTHILL

I. L. TYLER

WILLIAM A. MAPLES
Editor

ROBERT E. WILDE
Managing Editor

R. G. WIEDYKE
Assistant Editor

BARBARA BURESH
Editorial Secretary

Contributions to the Journal

The ACI JOURNAL technical pages are a medium for reporting what is of current interest in the concrete field; in research, design, construction, maintenance, or manufacturing. Members and readers are urged to "have their say" as a means of sharing knowledge and experience; time and changing interests demand that new names and new ideas take their places beside the old. Contributions describing new materials and techniques or new applications of familiar practices are welcomed by the Institute.

To guide the flow of material the Board of Direction has assigned to the Technical Activities Committee the selection of papers, committee reports, discussions, and other contributions for JOURNAL publication. To assist in their technical appraisal a corps of experts in various segments of the field volunteer their services. From the technical standpoint, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

General considerations by TAC are whether the contribution will be of interest and service, will fit into a well-balanced publications schedule, and can be published at reasonable expense to the Institute.

To expedite appraisal and publication, if accepted, contributors are requested to furnish glossy prints or inked tracings of illustrations accompanying manuscripts submitted for consideration. Further details of form and physical arrangement of contributions begin on p. 14, 1953 *ACI Directory*. Separate prints of the "publications policy" are available free upon request to the Secretary.

Manuscripts of papers, discussions, and reports should be sent in triplicate to:

Secretary, Technical Activities Committee

AMERICAN CONCRETE INSTITUTE

18263 W. McNICHOLS RD., DETROIT 19, MICHIGAN

In ACI Journal next month

"Effect of Curing on the Properties Affecting Shrinkage Cracking of Concrete Block," by J. C. Saemann, C. Warren, and G. W. Washa, reports on an investigation undertaken in an attempt to establish relationships between curing procedures and properties that influence shrinkage cracking of concrete block. Tests were made to determine modulus of rupture, compressive strength, tensile strength, modulus of elasticity in transverse bending, moisture volume changes, and thermal volume changes.

Karl V. Steinbrugge and Donald F. Moran offer "Performance of Reinforced Concrete and Concrete Masonry in Recent Western United States Earthquakes." Recent earthquakes have given an indication of the effectiveness of modern design methods and construction practices in resisting lateral forces. The examples of damage, although slight in some instances, are keys to weaknesses.

An investigation to determine whether a composite section acts monolithically with a straight line distribution of strain is presented in "Behavior of Prestressed Concrete Composite Beams," by R. H. Evans and A. S. Parker. Different types of prestressed elements were tested and the quality of bond for varying degrees of roughness of surface were observed.

In "Ball Test for Field Control of Concrete Consistency," by J. W. Kelly and Milos Polivka, a simple field test is described for determining the consistency of fresh concrete in terms of the penetration of a 6-in., 30-lb ball. The test was developed over a period of years, and is being extensively used throughout the country. Experiences of users are given, and comparisons with slump tests are made.

Tests on longitudinally reinforced concrete sections without web reinforcement are reported in "Strength of Reinforced Concrete T-Beams Under Combined Direct Shear and Torsion," by Earl I. Brown, II. The strength of reinforced concrete T-beams in combined direct shear and torsion are compared to the strength of similar sections in direct shear alone.

G. M. Bruere presents "Air Entrainment in Cement and Silica Pastes." The air-entraining capacities of a number of surface-active agents in cement and silica pastes were measured by a standard procedure. In addition, measurements were made of the foaming properties of surface-active agent solutions with and without electrolyte additions, and of the abilities of surface-active agents to make cement and silica particles hydrophobic by adsorption so that air bubbles adhere to them.

In "Eccentric Bending in Two Directions of Rectangular Concrete Columns," by Lu-Shien Hu, four charts are presented for checking the stresses in a rectangular reinforced concrete section subject to bending in two directions with or without normal compressive force. These charts can be used for sections with symmetrical as well as nonsymmetrical steel arrangements.

Title No. 51-35

Looking Ahead*

By CHARLES H. SCHOLER†

SYNOPSIS

Retiring ACI President Scholer looks ahead to the Institute's second 50 years. It is emphasized that part of the Institute's service to the concrete field is to reiterate the established principles of better concrete, not only to the new and unfamiliar, but especially to the established and eminent persons in the field.

THE SECOND 50 YEARS

A year ago the Institute completed its first 50 years of growth and development—half a century of inspiring progress, growing prestige, and increasing membership. The membership was proud to look back on the progress made by the child of its creation. We have now completed the first year of the second 50 years, and it is well for us to take stock of what accomplishments we might well hope to make in the latter half of our first century of development.

An old French proverb says, "The more things change, the more things are the same." In looking back over the scientific and technical developments in the field of concrete construction, the problems of the Institute faced in its growth and development from a mere handful of users to its present membership of about 7000, we should keep in mind that the really difficult things to achieve, the significant policies to be carried out, are substantially the same even though the means of executing these policies and projects may change. The vital element of the American Concrete Institute is still the sound integrity and ability of our membership, unselfishly devoting time and energy to the work of the Institute.

ACI AND THE CONCRETE INDUSTRY

The American Concrete Institute has had a strong, vigorous, healthy growth; it is still growing, and I believe is one of the soundest and best of many organizations of similar type, an organization where men interested in the technology of a given material or construction field meet together to discuss their common problems. We have a diversified membership. Without doubt, the majority of our members have some technical or engineering training, but by no means do all of our members have such training, and,

*Presented at the ACI 51st annual convention, Milwaukee, Wis., Feb. 23, 1955. Title No. 51-35 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Aug. 1, 1955. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Head, Applied Mechanics Department, Kansas State College, Manhattan, Kan.

in my judgment, much of the value of our organization depends on the wide variation in training, immediate interests, and particular tasks to perform in carrying out the aims of ACI. This unites our Institute into a sound, sensible, practical, needed organization in the development of our country. It is not devoted to the furtherance of the aims of any one group, but to the furtherance of real progress in our vital construction program.

All such organizations have two major problems to solve:

(1) The problems of presenting to the membership the new developments in the technology of the product.

(2) The problem of keeping the membership conscious of the well-known principles necessary for the proper utilization of the products, the principles so often overlooked and neglected.

Institute committee contributions

In presenting new developments, the committees of the Institute are performing their work in a very commendable manner. By no means are all of our problems solved. All 32 technical committees are continually presenting to the members of the Institute new developments and changes in practice for the education and guidance of all who work in the field of concrete design and construction. In some of these committees the major part of their work relates to new developments and research. Committees 115, Research; 209, Volume Changes and Plastic Flow in Concrete; 213, Properties of Lightweight Aggregates and Lightweight Aggregate Concrete; 215, Fatigue of Concrete; 323, Prestressed Reinforced Concrete; 327, Ultimate Load Design; 716, High-Pressure Steam Curing—bring together the best and most aggressive minds in the development of these fields, and present these facts to the members in an effective, workable manner. This aggressive program of work in new and rapidly developing fields should be continued. Other committees present primarily to the Institute well-known principles which change but slowly if at all but need to be brought together in a well formulated, standardized manner for the guidance of all engaged in concrete design and construction. The major committees in this field are the Standards Committee; Committee 207, Properties of Mass Concrete; 208, Bond Stress; 210, Resistance to Erosion in Hydraulic Structures; 212, Admixtures; 312, Plain and Reinforced Concrete Arches; 318, Standard Building Code; and 611, Inspection of Concrete. These committees are primarily presenting to the membership accepted standards of design and construction practices, practices with which most of the members are, or should be, rather familiar, but which any of us are likely to forget to follow.

Important contributions by ACI publications

Not only the committee work but the publication policies of the Institute tend to divide along these lines. All of us like to read papers describing new and daring designs, unique methods of solving unusual or difficult construction problems. Most of us, if we will stop to think about it, realize that our major difficulties do not rest in the unusual things we are doing;

our major difficulties are to maintain continuously a uniformly high standard of interest and care in carrying on the simple, everyday tasks that lie before us. The ultimate success of our over-all, marvelous construction program demands that these simple things be not neglected. In some way the Institute's publication policy should be such as to continually bring before its membership the serious need of thought and attention on common, ordinary, everyday matters; matters that involve really basic integrity and honesty; willingness to take pains in carrying on our program of work and supervising the work of others. I am afraid that these things are neglected fully as much in other fields of activity, including our country's political, economic, and social program, as well as in the design and construction program of the members of the Institute.

The Institute has several publications that emphasize the importance of these well-known, simple, homely virtues. The *Concrete Primer*, one of our old publications, now in process of being revised, should be re-read not only by the younger members of the Institute or men just entering the field of concrete design and construction, but particularly by eminent consulting engineers, administrative officials in high positions in major construction companies, and world-famous supervising architects. I am convinced that many of these people could profit immensely by such a perusal of what they are supposed to know. The ACI Building Code is a document of the utmost importance and value. The *Manual of Standard Practice for Detailing Reinforced Concrete Structures* is worthy of the attention not only of the humble engineer just out of school who is assigned the task of making detailed drawings of some engineer's design, but men much higher up in the organization may also review this with profit. A recent failure cited in engineering literature would have been avoided had the "Detailing Manual" been followed.

CONCRETE IMPROVEMENT GROUPS FORMED

This dissatisfaction with the standards of some of our construction problems and design practices is not restricted to college professors. Within the last few years, the unsatisfactory characteristics of much of our concrete construction in some of our greatest cities has drawn the attention of eminent men in the concrete construction field. These abuses and the failure to secure the best that is available have brought about the development of the Concrete Industries Board of New York City, and the Concrete Improvement Board of Detroit. I wish that I could say that these boards were organized not because some of the concrete was poor, but because concrete design and construction practices should be much better. I feel certain that these gentlemen are not alarmists or over critical of an industry that is really struggling to improve its product. The gruesome, ugly facts of human failure are there for anyone who cares to look to see them. Give these gentlemen credit for having seen it, and the courage to have said so. Also, I wish I could say that the conditions they observed are peculiar to these

great cities, but other cities have the same problem, and the smaller communities are faced with a somewhat similar problem.

Methods of concrete construction have developed until there is no excuse for poor concrete anywhere. Our portland cements are far more uniform and better than ever before in the history of the construction industry. The development of ready-mixed concrete has so improved proportioning, mixing, and handling practices that nonuniform concrete indicates only one thing; a failure of the human element. It is not a lack of adequate specifications, of good materials or good equipment, or a general pious desire for better concrete. It is a lack of understanding on the part, I am afraid, of men very high in the construction industry of the necessity and need to do the disagreeable, dirty work of weeding out those individuals who are not fundamentally honest and trying to produce a quality product. The men who organized these boards were men eminent in their profession who were concerned about a better quality concrete, a quality of concrete approaching what is now possible to readily produce. All concerned in the field of concrete construction should lend every assistance to them in carrying out their proposed project. Any way in which the American Concrete Institute can further this laudible enterprise should be undertaken. I do not believe that this will be a matter of rewriting our specifications, our code, or our inspection manual. It may consist of adopting such practices or doing such things as will bring about a better understanding of what is fundamentally wrong.

KEY MAN—THE INSPECTOR

On any construction project, the inspector or resident engineer is a key man in securing good construction. This man, in conjunction with the contractor, superintendent, and foremen are all that make any concrete project good or bad. The most marvelous specifications written by the most eminent engineer in the field of concrete design and construction are worthless unless their provisions are carried out in the field. Probably one of the weakest links in our whole program is a lack of respect and esteem accorded these men—the inspectors or resident engineers who make the decisions. It is here that the higher administrative officers in the engineering firm, the supervising architect, or the general contractor can do their most effective work in promoting better concrete. Not only should these men be well trained, and well paid, but they should be given adequate consideration and prestige. They should be openly recognized as men who, after all, are the ones who make the decisions and determine whether or not the best results will be secured.

RESPONSIBILITIES OF INDUSTRY LEADERS

In too many places, even in Kansas, specifications developed by many of our engineers and architects provide that the costs of inspection and testing are to be borne by the general contractor. Personally, I think this is an unethical practice, not fair to the owner whose interests the engineer and

architect are ethically bound to protect. Such fees and costs are just as legitimate an item of cost to the owner as are the fees of the engineer or the architect. Too many eminent engineers who would stand in horror at the thought of competitive bidding for engineering services shop around for the cheapest testing and inspection service they can find.

It is for these reasons that I state that the major problem in getting better concrete is to have a real sincere, deep-seated desire on the part of these eminent men to secure better concrete. It is not fair to blame the inspector for this work. Too often he has attempted to remedy bad construction practices or to reject unsatisfactory materials which would not meet the specifications, and when this was brought to the attention of the engineer and architect in charge, the whole matter was hushed up and brushed aside. Political and financial pressure upon his superiors, the fear that they may offend some one, and have future difficulties on future contracts, caused his recommendation to be ignored. Too often he learns this from the general contractor. After all, he is a man trying to make a living, and when he learns that he is most appreciated when he causes the least amount of trouble and dissension, that he gets his pay from the general contractor, and is most appreciated when he does nothing, he soon subsides to the more or less automatic mouthpiece some people like to use. This condition cannot be corrected until men of the caliber who are now heading these improvement boards are willing to take some risks, risks of displeasing the wrong persons or failing to be in touch with the right group in the right way. Such risks may be temporarily disagreeable and costly, but in the long run they can raise the standards of concrete construction performance; nothing else can bring about such a change; it can be done at no added expense.

Again, let me repeat that if there is a sad and unfortunate condition in our industry, it is the fault of many people who have not been willing to fully do their duty. It is a human failure, the kind you and I are very likely to be guilty of committing or permitting to develop under our supervision. It is a practice we can stop if we have the desire and courage of our convictions.

Title No. 51-36

Theory of the Secondary Arch*

By B. F. JAKOBSEN†

SYNOPSIS

For circular arches having a ratio of thickness to mean radius greater than 0.3, a central angle less than 100 deg, and under normal loads, high tensile stresses are indicated both at the crown and at the abutments by the usual formulas. A method of computation is proposed which assumes that the stress distribution is linear on radial sections and that the part of the arch not subject to compression may be disregarded.

INTRODUCTION

Investigations of the stresses in a circular arch under normal loads show that for arches having a ratio of thickness to mean radius greater than 0.3 and a central angle less than 100 deg high tensile stresses are indicated by the usual formulas both at the crown and at the abutments.[1] For example, for an arch having a ratio of $t/r = 0.4$ and a central angle of 90 deg, subject to 200 ft of water pressure and a temperature drop of 30 F, computations show that at the abutments there would be a tension of 650 psi at the extrados and a compression of 1170 psi at the intrados.[2] Since such tensile stresses cannot exist in an arch dam as usually constructed, recourse must be had to a secondary arch free from tension to estimate the actual compressive stresses involved. A method of computing the dimensions and the stresses in a secondary arch of constant thickness drawn inside the primary arch has been given[3] but this is satisfactory only when the dimensions of the secondary arch are nearly the same as those of the actual arch.

The method of computation proposed here assumes that the stress distribution is linear on radial sections and that the part of the arch not subject to compression may be disregarded. As a consequence the half-arch is divided into three segments (Fig. 1) designated as crown, middle, and abutment segments.

*Received by the Institute Oct. 5, 1953. Title No. 51-36 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Aug. 1, 1955. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Consulting Engineer, Menlo Park, Calif.

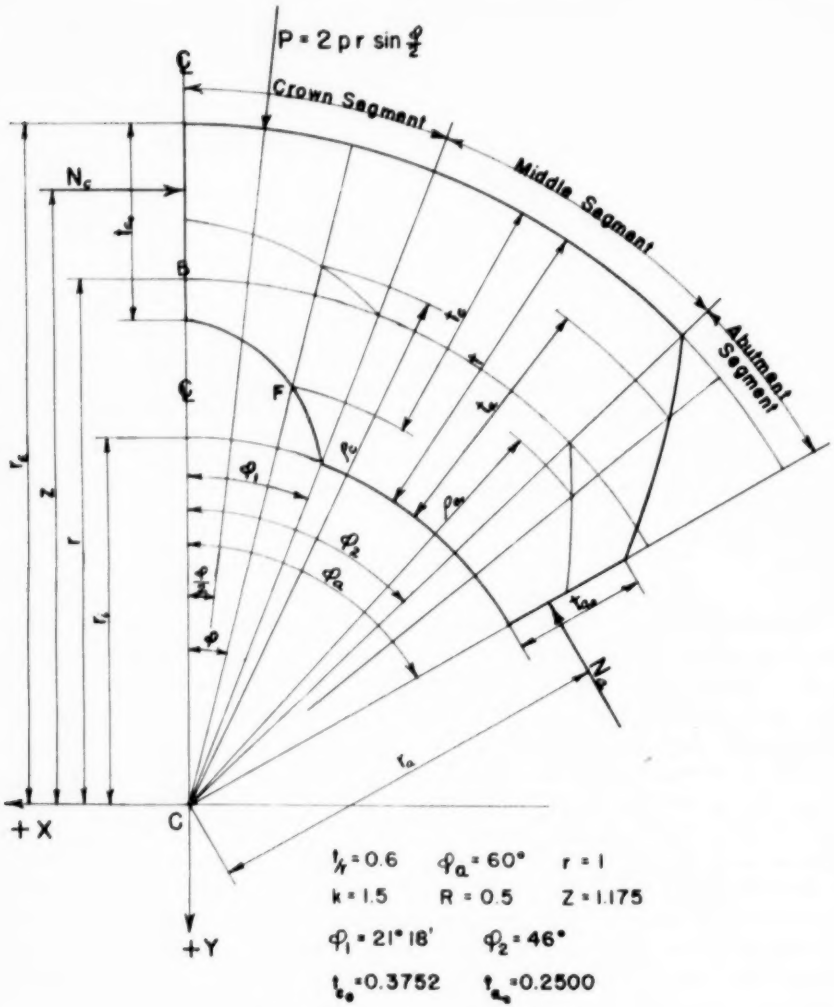


Fig. 1—Segmental division of half-arch

NOTATION

M = bending moment for a slice of arch 1 ft thick normal to the plane of Fig. 1, positive when it tends to produce tension in the upstream face, ft-lb

θ = angle of rotation, positive in the same direction as M, *i.e.*, counter-clockwise in Fig. 1, radians

N = normal force on a slice of arch 1 ft thick, positive when in compression, lb

S = shear force on a slice of arch 1 ft thick, positive when directed toward the center C of the arch, lb

X = displacement of the midpoint of the crown section (Point B in Fig. 1), positive when in the direction of the positive x axis, ft

r = mean radius, ft
t = radial thickness, ft
p_e = water pressure on upstream face, psf
p = water pressure referred to the centerline of the arch and equal to $p_e r_e/r$, psf
E and E_f = modulus of elasticity for concrete and foundation rock, respectively, psf
The significance of r_e, r_a, r_i, z, t_c, t_a, and Θ_c is shown in Fig. 1.

GENERAL FORMULAS

Consider a 1 ft thick slice of an arch cut out of an arch dam and prevented from moving perpendicular to its plane.

The reactions produced by the water pressure p are N_c acting at the crown section a distance z from the center C of the arch, N_a acting at the abutment a distance r_a from the center C, and the shear force S_a at the abutment. The stresses and deformations are completely determined when these five quantities are known. The three equilibrium equations are, in the direction of N_a:

$$N_a = N_c \cos \phi_a + P \sin (\tfrac{1}{2}\phi_a) \quad \ldots\ldots (1)$$

In the direction of S_a:

$$-S_a = N_c \sin \phi_a - P \cos (\tfrac{1}{2}\phi_a) \quad \ldots\ldots (2)$$

in which $P = 2pr \sin (\phi_a/2)$ is the total water pressure, and the moments about the center C of the arch are

$$N_c z = N_a r_a \quad \ldots\ldots (3)$$

Combining Eq. (1) and (3) gives

$$N_c = \frac{1 - \cos \phi_a}{(z/r_a) - \cos \phi_a} pr \quad \ldots\ldots (4)$$

The ratio z/r_a must be so determined that the external forces produce no rotation at the crown section, *i.e.*,

$$\Sigma \theta = 0 \quad \ldots\ldots (I)$$

N_c and N_a are both proportional to pr and so is X_p, the displacement of the midpoint of the crown section in the direction of the x axis due to the water pressure. This displacement must compensate for the displacement X_T caused by the temperature variation occurring after closure of the arch, or

$$X_p + X_T = 0 \quad \ldots\ldots (II)$$

The procedure is to assume z and $k = z/r_a$ and compute the rotation of the crown section; if this is not zero or nearly so, a new value of z is assumed and another computation made with the same value of k. If the rotation obtained from Eq. (I) is positive, z must be increased and vice versa. When corresponding values of z and k have been found satisfying Eq. (I), a relation between water load and temperature variation is obtained from Eq. (II). It has been found that corresponding values of z and k which satisfy Eq. (I) lie close to a straight line through two points, $z = r_e$, $r_a = r_i$ and $z = r + t/6$, $r_a = r - t/6$. The equation of this line is

$$r_a = 2r - z \quad \ldots\ldots (III)$$

It may be here noted, that in the actual computations r is assumed equal to 1 and t, r_e, r_a, and ρ_c are all expressed in terms of $r = 1$.

The normal force N, the bending moment M, and the shear force S acting on a radial section (Fig. 2) are

$$\left.\begin{aligned} N &= N_c \cos\phi + pr\,(1 - \cos\phi) \\ M &= pr\rho(1 - \cos\phi) - N_c(z - \rho\cos\phi) \\ S &= (pr - N_c)\sin\phi \end{aligned}\right\} \quad \ldots\ldots(5)$$

The rotation of the crown section resulting from the deformation of the arch, not including the yielding of the foundation, is due to M only and is,

$$E\Theta = \Sigma(M/I)\,\rho\Delta\phi \quad \ldots\ldots(6)$$

in which $I = t^3/12$ and t is the effective thickness of the arch; this assumes that the neutral axis and the centerline of the arch coincide, which is approximately true.[4] The displacements in the direction of the x axis due to N, M, and S are, respectively,

$$\left.\begin{aligned} EX_N &= -\Sigma(N/t)\cos\phi\,\rho\Delta\phi \\ EX_M &= \Sigma(M/I)\,(r - \rho\cos\phi)\,\rho\Delta\phi \\ EX_S &= 2.88\,\Sigma(S/t)\sin\phi\,\rho\Delta\phi \end{aligned}\right\} \quad \ldots\ldots(7)$$

The summation extends from $\phi = 0$ to ϕ_a. The coefficient $2.88 = (1.2)(2.4)$ in which the first factor is due to the assumption of parabolic distribution of shear stress and $2.4 = E/G = 2(m + 1)/m$ in which G is the shear modulus and $1/m$ is Poisson's ratio assumed to be $1/5$ or $m = 5$.

To simplify the equations let

$$k = z/r_a = N_a/N_c;\ R = N_c/pr = N_a/k\,pr \text{ and } u = N/pr \quad \ldots\ldots(8)$$

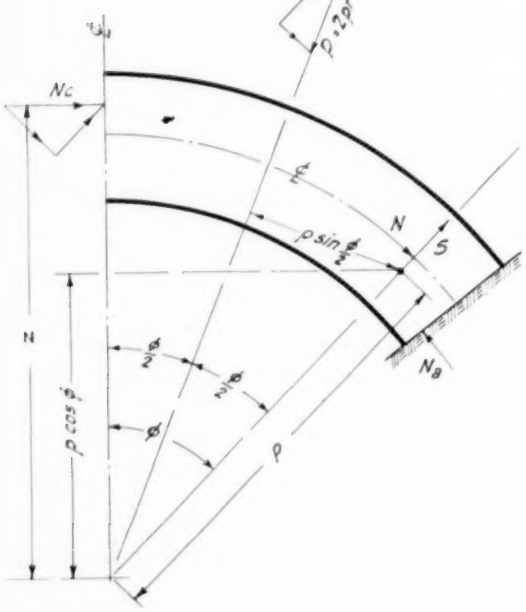


Fig. 2—Normal force, bending moment and shear acting on a radial section

From the first Eq. (5) it follows that

$$u = 1 - (1 - R)\cos\phi \quad (9)$$

and for $\phi = \phi_a$, dividing by pr and arranging,

$$R = \frac{1 - \cos\phi_a}{k - \cos\phi_a} \quad (9a)$$

For a given arch R depends on k only.

The length of the arch is decreased by a drop of temperature and therefore z and k are greater than 1 and R and r_a less than 1. For $\phi = 0$, $u = R = N_c/pr$ and for $\phi = \phi_a$, $u = kR = N_a/pr$. Introducing R and u into Eq. (5) gives

$$\left.\begin{aligned} N &= pru \\ M &= (\rho u - Rz)pr \\ S &= (1 - R)\,pr\sin\phi \end{aligned}\right\} \quad (10)$$

The location of the point of application of N is given by $N r_a = N_c z$, or

$$r_a = R\,z/u \quad (11)$$

The angles ϕ_1 and ϕ_2 of the secondary arch are obtained by putting $r_a = r + t/6$ and $r_a = r - t/6$, respectively, so that

$$\left.\begin{aligned} (1 - R)\cos\phi_1 &= 1 - 6Rz/(6r + t) \\ (1 - R)\cos\phi_2 &= 1 - 6Rz/(6r - t) \end{aligned}\right\} \quad (12)$$

ROTATIONS AND DISPLACEMENTS

Crown segment, $\phi = 0$ to ϕ_1

The rotation of the crown section caused by the deformation of the crown segment is obtained from Eq. (6). The thickness and the mean radius of the effective arch are respectively, $t_c = 3(r_a - r_a)$ and $\rho_c = r_a - t_c/6$. Let $c = Rz/r_a$

Then

$$t_c = 3\,r_a\frac{u - c}{u} \text{ and } \rho_c = \tfrac{1}{2}\frac{3c - u}{u}\,r_a \quad (13)$$

Introducing these expressions into Eq. (6) and arranging,

$$E\Theta_c = -\frac{pr}{9r_a}\sum\frac{3c - u}{(u - c)^2}\,u^2\,\Delta\phi$$

or more conveniently for computations

$$E\Theta_c = -\frac{pr}{9r_a}\sum\left[-(u - c) + c^2\left(\frac{3}{u - c} + \frac{2c}{(u - c)^2}\right)\right]\Delta\phi \quad (14)$$

The displacements in the direction of the positive x axis of the midpoint of the crown section of the primary arch produced by N, M, and S are obtained from Eq. (13) and (7). They are

$$\left.\begin{aligned} EX_{cN} &= -\frac{1}{6}\,pr\sum\left(2c - u + \frac{2c^2}{u - c}\right)\cos\phi\,\Delta\phi \\ EX_{cM} &= E\Theta_c r + \frac{pr}{18}\sum\left(u - 4c + \frac{4c^3}{(u - c^2}\right)\cos\phi\,\Delta\phi \\ EX_{cS} &= 0.48(1 - R)\,pr\sum\left(\frac{2c}{u - c} - 1\right)\sin^2\phi\,\Delta\phi \end{aligned}\right\} \quad (15)$$

Summation from $\phi = 0$ to ϕ_1

Eq. (14) and (15) may be more conveniently computed from the integrals listed below. For explanation of methods of integration see the appendix.

Let $A = 1 - c = 1 - Rz/r_s$, $B = -(1 - R)$ and $G^2 = (A - B)/(A + B) > 0$. Then

$$
\left.
\begin{aligned}
I_{ci} &= \int_0^{\phi_1} \frac{1}{u - c}\,d\phi = \frac{2G}{A - B} \arctan\left[G \tan\left(\tfrac{1}{2}\phi_1\right)\right] \\
I_{cii} &= \int_0^{\phi_1} \frac{d\phi}{(u - c)^2} = \frac{1}{(A + B)(A - B)}\left[A\, I_{ci} - \frac{B \sin\phi_1}{A + B\cos\phi_1}\right] \\
I_{cN} &= \int_0^{\phi_1} \frac{\cos\phi}{u - c}\,d\phi = \frac{1}{B}\left(\phi_1 - A\, I_{ci}\right) \\
I_{cM} &= \int_0^{\phi_1} \frac{\cos\phi\, d\phi}{(u - c)^2} = \frac{1}{B}\left(I_{ci} - A\, I_{cii}\right) \\
I_{cS} &= \int_0^{\phi_1} \frac{\sin^2\phi\, d\phi}{u - c} = \frac{1}{B^2}\left[A\,\phi_1 - B \sin\phi_1 - (A + B)(A - B)\, I_{ci}\right]
\end{aligned}
\right\} \quad \ldots\ldots (15a)
$$

Middle segment, $\phi = \phi_1$ to ϕ_2

The thickness of the effective arch is constant and equal to t and $\rho = r$, so that Eq. (6), and (7) may be readily integrated, giving

$$
\left.
\begin{aligned}
E\,\Theta_m &= 12\,p\,(r/t)^3\left[(1 - Rz)\phi - (1 - R)\sin\phi\right]\Big|_{\phi_1}^{\phi_2} \\
E\,X_{mN} &= -\frac{p\,r^2}{t}\left[\sin\phi - \frac{1 - R}{2}\left(\phi + \frac{\sin 2\phi}{2}\right)\right]\Big|_{\phi_1}^{\phi_2} \\
E\,X_{mM} &= E\,\Theta_m r - 12\,pr(r/t)^3\left[(1 - Rz)\sin\phi - \frac{1 - R}{2}\left(\phi + \frac{\sin 2\phi}{2}\right)\right]\Big|_{\phi_1}^{\phi_2} \\
E\,X_{mS} &= 1.44\,\frac{pr^2}{t}(1 - R)\left(\phi - \frac{\sin 2\phi}{2}\right)\Big|_{\phi_1}^{\phi_2}
\end{aligned}
\right\} \quad \ldots (16)
$$

These are the rotation and displacements in the direction of the positive x axis of the midpoint of the crown section caused by the deformation of the middle segment.

Abutment segment, $\phi = \phi_2$ to ϕ_a

The thickness and the mean radius are, respectively, $t_a = 3\,(r_o - r_i)$ and $\rho_a = r_o + t_a/6$.

Let

$$a = R\,z/r_i$$

Then

$$t_a = 3\,r_i\,\frac{a - u}{u} \text{ and } \rho_a = \tfrac{1}{2}\,r_i\,\frac{3\,a - u}{u} \quad \ldots\ldots (17)$$

Introducing these expressions into Eq. (6) and (7) and arranging

$$
\begin{aligned}
E\,\Theta_a &= \frac{pr}{9r_i}\sum\left[(a-u)+a^2\left(-\frac{3}{a-u}+\frac{2a}{(a-u)^2}\right)\right]\Delta\phi \\
E\,X_{aN} &= -\tfrac{1}{6}\,pr\sum\left[u-2a+\frac{2a^2}{a-u}\right]\cos\phi\,\Delta\phi \\
E\,X_{aM} &= E\,\Theta_a r - \frac{pr}{18}\sum\left(u-4a+\frac{4a^2}{(a-u)^2}\right)\cos\phi\,\Delta\phi \\
E\,X_{aS} &= 0.48\,(1-R)\,pr\sum\left(\frac{2a}{a-u}+1\right)\sin^2\phi\,\Delta\phi
\end{aligned}
\qquad \ldots(18)
$$

These are the rotation and displacements of point B in Fig. 1, resulting from the deformation of the abutment segment.

The following integrals apply to Eq. (18).

Let $C = a - 1 = Rz/r_i - 1 < 0$, $D = 1 - R > 0$ and $H^2 = -(C + D)/(C - D)$, so that $H > 0$, then,

$$
\begin{aligned}
I_{ai} &= \int_{\phi_2}^{\phi_a}\frac{d\phi}{a-u} = \frac{-2.303}{(C-D)\,H}\,Log\left\{\frac{H+\tan\frac{\phi_a}{2}}{H-\tan\frac{\phi_a}{2}}\times\frac{H-\tan\frac{\phi_2}{2}}{H+\tan\frac{\phi_2}{2}}\right\} \\
I_{aii} &= \int_{\phi_2}^{\phi_a}\frac{d\phi}{(a-u)^2} = \frac{-1}{(C-D)(C+D)}\left[\frac{D\sin\phi_a}{C+D\cos\phi_a}-\frac{D\sin\phi_2}{C+D\cos\phi_2}-C\,I_{ai}\right] \\
I_{aN} &= \int_{\phi_2}^{\phi_a}\frac{\cos\phi\,d\phi}{a-u} = \frac{1}{D}\left[\phi_a-\phi_2-C\,I_{ai}\right] \\
I_{aM} &= \int_{\phi_2}^{\phi_a}\frac{\cos\phi\,d\phi}{(a-u)^2} = \frac{1}{D}\left[I_{ai}-C\,I_{aii}\right] \\
I_{aS} &= \int_{\phi_2}^{\phi_a}\frac{\sin^2\phi}{a-u} = \frac{1}{D^2}\left[C(\phi_a-\phi_2)-D(\sin\phi_a-\sin\phi_2)-(C-D)(C+D)I_{ai}\right]
\end{aligned}
\qquad \ldots(18a)
$$

Foundation

The rotation of the crown section and the displacement of the midpoint of the crown section caused by the deformation of the foundation is obtained from the formulas of Vogt.[5,6]

The rotation is

$$E_f\,\Theta_f = \alpha\,M_a/t^2_{ao} + \beta\,S_a/t_{ao}$$

The displacement parallel to the abutment, *i.e.*, radially toward the center C of the arch is

$$E_f\,V_f = \gamma\,S_a + \beta\,M_a/t_{ao}$$

The displacement normal and into the abutment is

$$E_f\,U_f = \gamma\,N_a$$

M_a, N_a, and S_a are the moment, compression, and shear at the abutment and $t_{ao} = 3\,(r_a - r_i)$ is the radial thickness of the effective arch at the abutment. Vogt gives the following average values of the coefficients, $\alpha = 5.14$ to 5.53; $\beta = 0.57$ to 0.79 and $\gamma = 1.40$ to 2.14 depending upon the dimensions

and shape of the loaded surface and on Poisson's ratio. Taking the average values of the coefficients, the rotation and the displacements of the crown section when $t_{ao} < t$ are,

$$E\,\Theta_f = \frac{0.294}{r_a - r_i}\,[kR + 0.77\,(1 - R)\sin\phi_a]\,pr\,E/E_f \quad \ldots\ldots (19)$$

$$\left.\begin{aligned} E\,U_f &= 1.8\,kR\,pr\,E/E_f \\ E\,V_f &= [1.8\,(1 - R)\sin\phi_a + 0.11\,kR]\,pr\,E/E_f \\ E\,X_f &= E\,V_f \sin\phi_a - E\,U_f\cos\phi_a + (r - \rho_{ao}\cos\phi_a)\,E\,\Theta_f \end{aligned}\right\} \quad \ldots\ldots (20)$$

in which $\rho_{ao} = 1/2\,(3\,r_a - r_i) < r$.

Selecting a correct value for E_f is obviously difficult. In addition to the elastic deformations there may be permanent displacements caused by joints tightening up. Generally EX_f is positive and tends to decrease the maximum stresses, so it would be in the interest of safety to select E_f too large.

It may be noted that since $z > r$, $E\Theta_c$, EX_{cM} and EY_{cM} are negative, while $E\Theta_a$, EX_{aM} and EY_{aM} are positive.

APPLICATION OF FORMULAS

A value of z greater than 1 is assumed and r_a computed from Eq. (III); then $k = z/r_a$. R and u are obtained from Eq. (9) and r_a from Eq. (11). The limiting angles of the segments ϕ_1 and ϕ_2 are computed from Eq. (12). For the computations of $E\Theta_c$, the constant c is given by Eq. (13) and if the summation formula, Eq. (14), is to be used, $1/(u - c)$ and $1/(u - c)^2$ are computed for several values of ϕ between zero and ϕ_1. If the integral formula is to be used, I_{ci} and I_{cii} are found from Eq. (15a) and inserted in Eq. (14). The integral formulas involve less work and are more accurate, but the summation formulas are simpler and if an error is committed, it is more likely to be noticed by inspection or by plotting the individual values of $1/(u - c)$ and $1/(u - c)^2$.

If the total rotation is not zero or nearly so, a new value of z is selected, keeping k, R, and u the same as before; the new value of r_a is given by z/k. If the rotation was positive, $\Sigma\,E\Theta > 0$, then z must be increased and vice versa. The new computations start by determining the limiting angles ϕ_1 and ϕ_2 and then proceed as outlined above. When a value of z has been found for the assumed value of k which makes the total rotation zero, the displacements can be found. The sum of all displacements caused by the water pressure is X_p.

A temperature variation produces a displacement of the crown section, $X_T = c\,\Delta T\,r\sin\phi_a$ in which c is the temperature constant of the concrete and ΔT the variation of temperature after closure of the arch, positive for a temperature rise. Since the crown displacement must be zero,

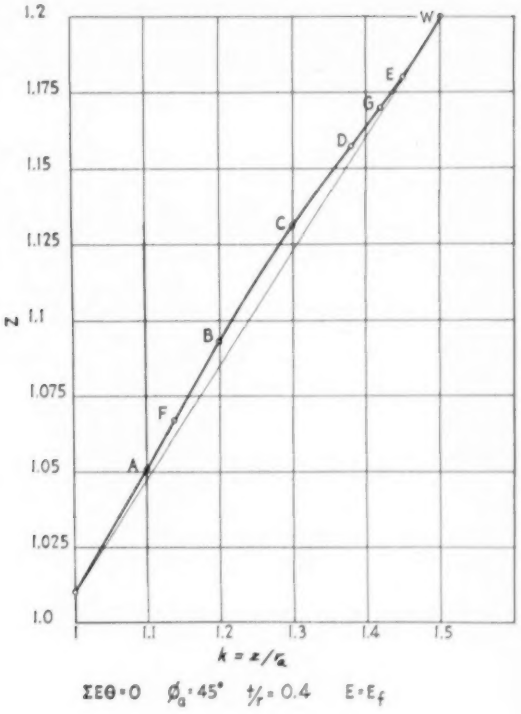
$$E\,X_p + E\,X_T = 0 \quad \ldots\ldots (II)$$

or

$$E\,X_p = C(pr) = -\,E\,X_T = -\,c\,\Delta T\,E\,r\sin\phi_a \quad \ldots\ldots (21)$$

so that

Fig. 3—Variations of k



$$\Delta T = \frac{-C}{c\,E\,r \sin \phi_a}\, pr$$

ΔT is the temperature variation required to compensate for the X displacement of the crown produced by the water pressure. $C = EX_p/pr$ is a constant for any given secondary arch and depends only on z and k. Usually ΔT is negative and C must be positive and it decreases as pr increases.

As an example consider an arch for which $t/r = 0.4$ and the half central angle $\phi_a = 45$ deg and with $E = E_f$. A number of corresponding values of z, k, and $r_a = z/k$ were computed for which the rotation of the crown section was zero or negligible. The resulting values of z and k are plotted as points A, B, C, ... in Fig. 3. Point A belongs to the primary arch and point F marks the end of the primary arch and the beginning of the secondary arch, *i.e.*, the point where either N_c or N_a lie at the edge of the middle third. Point A was computed to show continuity and to serve as a check on the computations. The values of the displacements $EX_p = Cpr$ of the crown section for the points A, B, C, ... were then computed and plotted in Fig. 4 against z. To fix the relation between water pressure and temperature variation it is necessary to assume the stress at some point of the arch and for this purpose the maximum stress, which occurs at the intrados of the abutment, was selected, and

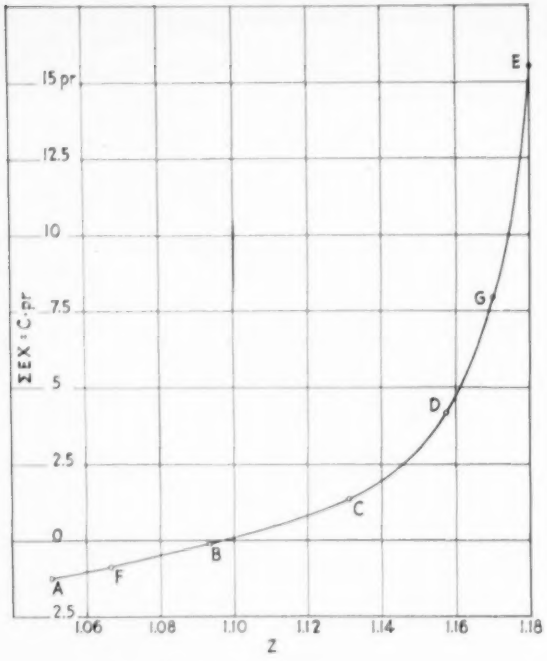


Fig. 4—Values of displacement of crown section in relation to z

$$n_{ao} = 2\,N_a/l_{ao} = 2\,kR\;pr/3(r_a - r_s) \quad \ldots\ldots (22)$$

For N_a and R see Eq. (8) and (9). The constants z, k, r_a, etc., for the points B, C, D, ... are listed in Table 1.

In Fig. 5 the relation between head of water and temperature variation is plotted for $n_{ao} = 1200$ psi and $c\,E = 5.5 \times 300$. The computations for point C are as follows. From Eq. (22), $n_{ao} = 2 \times 1.3 \times 0.4940\;pr/3(0.8700 - 0.8000) = 6.116\;p$ and consequently, $p = 1200 \times 144/6.116 = 28{,}250$ psf and the corresponding head is $h = 28{,}250/1.2 \times 62.5 = 377$ ft. The corresponding temperature drop is obtained from Eq. (21), $C\;pr = 1.40\;p = -5.5 \times 300 \times 0.7071\;\Delta T$ or $\Delta T = -34$ deg.

Fig. 5 shows that for $n_{ao} = 1200$ psi and a 20 deg temperature drop, $h = 440$ ft, while if the temperature drop is 40 deg the head $h = 340$ ft. The advantage of keeping the temperature drop as small as economically feasible is obvious.

TABLE 1—CONSTANTS

	z	k	$r_a = z/k$	R	$EX_p = C\;pr$	$z + r_a$	ΔT, deg
B	1.094	1.20	0.9117	0.5942	−0.144 pr	2.0057	+ 5
C	1.131	1.30	0.8700	0.4940	+1.400 pr	2.0010	−34
D	1.158	1.38	0.8391	0.4353	4.200 pr	1.9971	−61
G	1.170	1.42	0.8239	0.4109	8.000 pr	1.9939	−74
E	1.180	1.45	0.8138	0.3953	15.600 pr	1.9983	−84

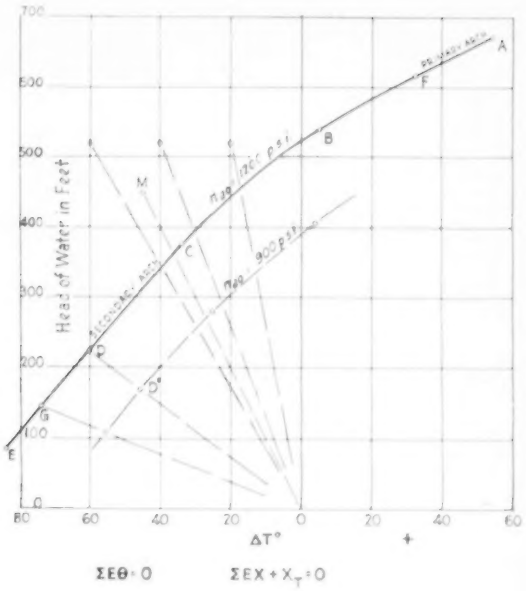


Fig. 5—Relation between head of water and temperature variation

To illustrate the application of the diagrams, suppose the arch is subject to a head of 450 ft and a temperature drop of 45 F (point M in Fig. 5). Then $p = 450 \times 62.5 \times 1.2 = 33{,}750$ psf and from Eq. (21), $C = 5.5 \times 300 \times 0.7071 \times 45/33{,}750 = 1.56$ so that from Fig. 4, $z = 1.134$ and from Fig. 3, $k = 1.31$ and consequently $r_a = z/k = 0.866$; R from Eq. (9) is $(1 - 0.7071)/(1.31 - 0.7071) = 0.486$ and the maximum stress is $n_{ao} = 2 \times 1.31 \times 0.486\ pr/3 \times 0.066 = 6.43 \times 33{,}750$ psf or 1510 psi.

Fig. 5 is plotted for $n_{ao} = 1200$ psi. Curves for other values of n_{ao} may be obtained by simple proportions. Consider for example point D. Eq. (21) and (22) show that if both p and ΔT are reduced say to 75 percent of their value in Fig. 5, then $C = EX_p/pr$ remains constant and therefore also k, R, and t_{ao} and consequently n_{ao} is also reduced to 75 percent of its value. The new point D' is found by making $OD' = 0.75\ OD$. Likewise if $h = 450$ ft and $\Delta T = -45$ deg, *i.e.*, point M in Fig. 5, the distance OM is found to be 1.25 times the distance along OM from 0 to the curve $n_{ao} = 1200$ and for point M accordingly $n'_{ao} = 1.25 \times 1200 = 1500$ psi as found above by computation.

If the arch cools under a constant head of $h = 520$ ft with $n_{ao} = 1200$ for $\Delta T = 0$, then for $\Delta T = -20$ deg, -40 deg, and -60 deg, $n_{ao} = 1370$, 1560, and 1820, respectively. While the stress increases, N_c decreases.

Fig. 5 therefore will give the stress n_{ao} for any combination of h and ΔT likely to be of interest. The shape of the secondary arch for point C in Fig. 5, is shown to scale in Fig. 6.

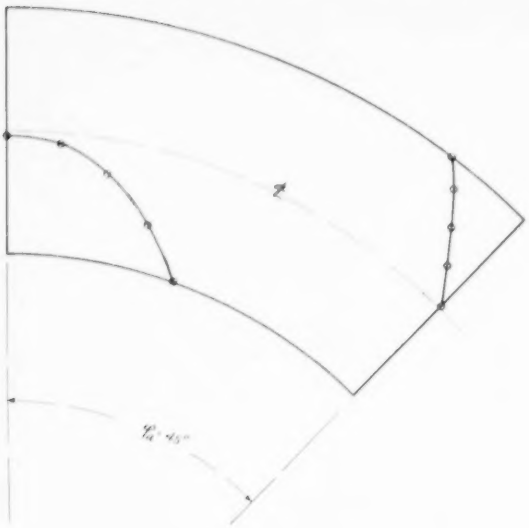


Fig. 6—Shape of secondary arch for point C in Fig. 5

MAXIMUM STRESSES

The stresses at the extrados of the crown and at the intrados of the abutment are, respectively, $n_{co} = 2N_c/t_{co} = 2R\ pr/3(r_e - z)$ and $n_{ao} = 2N_a\ pr/3(r_a - r_i)$ and their ratio is $n_{ao}/n_{co} = k(r_e - z)/(r_a - r_i)$. With the approximation of Eq. (III), $z = 2r - r_a$, the ratio equals k. This means that n_{ao} may be some 30 or 40 percent greater than n_{co}, so that if n_{ao} can be reduced a considerable saving may be effected. As an illustration consider the point D in Fig. 5 (see also Table 1). The maximum stresses at the crown and at the abutment are respectively $n_{co} = 6.91\ pr$ and $n_{ao} = 10.23\ pr$, a ratio of 1.48 or somewhat greater than k. Computations show that at $\phi = 43$ deg the maximum stress is only 6.67 pr or less than the maximum stress at the crown. If $r = 200$ ft, the 2 deg represent only about 7 ft at each abutment. n_{ao} can probably be reduced by thickening the arch slightly at the abutment, or as an alternative, it may be possible to use a somewhat richer mix at the abutment. At $\phi_2 = 35$ deg 28 min the maximum stress is only about 2.7 pr.

Y DISPLACEMENTS

These displacements may be computed from equations similar to Eq. (7); they are,

$$\left.\begin{aligned} E\,Y_N &= \Sigma(N/t)\sin\phi\,\rho\,\Delta\phi \\ E\,Y_M &= \Sigma(M/I)\sin\phi\,\rho^2\,\Delta\phi \\ E\,Y_S &= 2.88\,\Sigma(S/t)\cos\phi\,\rho\,\Delta\phi \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Since the deflections for a number of points may be wanted, the computations can probably be most conveniently made by summation, except for the middle segment. The following equations were derived from Eq. (23).

Crown segment, $\phi = 0$ to ϕ_1

$$\left.\begin{aligned} E\,Y_{cN} &= \frac{pr}{6}\sum\left[2c - u + \frac{2c^2}{u-c}\right]\sin\phi\,\Delta\phi \\ E\,Y_{cM} &= -\frac{pr}{18}\sum\left[u - 4c + \frac{4c^3}{(u-c)^2}\right]\sin\phi\,\Delta\phi \\ E\,Y_{cS} &= 0.48\,(1-R)\,pr\sum\left[\frac{2c}{u-c} - 1\right]\sin\phi\cos\phi\,\Delta\phi \end{aligned}\right\} \quad (24)$$

Middle segment, $\phi = \phi_1$ to ϕ_2

$$\left.\begin{aligned} E\,Y_{mN} &= -\frac{pr^2}{t}\left[\cos\phi_2 - \cos\phi_1 + \frac{1-R}{2}(\sin^2\phi_2 - \sin^2\phi_1)\right] \\ E\,Y_{mM} &= -12\,pr\left(\frac{r}{t}\right)^2\left[(1-Rz)(\cos\phi_2 - \cos\phi_1) + \frac{1-R}{2}(\sin^2\phi_2 - \sin^2\phi_1)\right] \\ E\,Y_{mS} &= 1.44\,pr\left(\frac{r}{t}\right)(1-R)\,(\sin^2\phi_2 - \sin^2\phi_1) \end{aligned}\right\} \quad (25)$$

Abutment segment, $\phi = \phi_2$ to ϕ_a

$$\left.\begin{aligned} E\,Y_{aN} &= \frac{pr}{6}\sum\left[u - 2a + \frac{2a^2}{a-u}\right]\sin\phi\,\Delta\phi \\ E\,Y_{aM} &= \frac{pr}{18}\sum\left[u - 4a + \frac{4a^3}{(a-u)^2}\right]\sin\phi\,\Delta\phi \\ E\,Y_{aS} &= 0.48\,(1-R)\,pr\sum\left[\frac{2a}{a-u} + 1\right]\sin\phi\cos\phi\,\Delta\phi \end{aligned}\right\} \quad (26)$$

The Y displacement of the crown section due to the deformation of the foundation is

$$E\,Y_f = E\,V\cos\phi_a + E\,U\sin\phi_a + \rho_{ao}\sin\phi_a\,E\,O_f \quad (27)$$

For V, U, and ρ_{ao} see Eq. (20).

The displacement produced by a temperature variation is given by

$$E\,Y_T = -c\,T\,E\,r\cos\phi_a \quad (28)$$

in which c is the temperature coefficient of the concrete.

The Y displacements given above are for the midpoint of the crown section. If the displacement of another point on the centerline of the primary arch is wanted, it is necessary to diminish the Y displacements accordingly. For example, the displacement due to moments of the midpoint of the radial section between crown and middle segments would be

$$EY_M = EY_{mM} - EO_m r\sin\phi_1 + EY_{aM} - EO_a r\sin\phi_1 + EY_f - r\sin\phi_1\,EO_f$$

In an arch dam the load on the arches will be decreased by the load carried by the cantilevers. This is generally greater at the abutments than at the crown. It would appear that the arches could safely be designed for a uniform load equal to the full water load minus the load taken by the cantilevers at the crown. To determine the maximum water load which can safely be carried by the cantilevers, the writer would start at the base as-

suming full water pressure on the cantilevers and above that base limit the reduction of arch load to the load the cantilevers can carry without tension in the upstream face, no matter what the deflections indicate. This would take account of the fact that the margin of safety of a gravity section with zero compression at the upstream face is negligible and precarious, while that of an arch is very large.

GENERAL REMARKS

The stress determination depends on the ratio E/E_f for determining the abutment deformation. This ratio is likely to vary considerably along the abutments and at best can only be estimated with the probability of a considerable error. To give an idea of the influence of such an error on the rotation and displacements, the following values are given for point C in Fig. 5. The rotations are: $\Theta_c = -5.6433\ p/E$; $\Theta_m = -0.054\ p/E$; $\Theta_a = +1.8282\ p/E$ and $\Theta_f = +3.854\ p/E$, so that $\Sigma\Theta = -0.0151\ p/E$. The X displacements are: $X_c = -0.2589\ pr/E$; $X_m = -0.0547\ pr/E$; $X_a = +0.6346\ pr/E$ and $X_f = +1.07566\ pr/E$, so that $X_p = +1.3967\ pr/E$. As will be noted, the influence of the abutment deformation is quite considerable. In this case $E = E_f$ was assumed and generally it is believed E_f will be larger and sometimes much larger than E, which would mini.nize the influence of the abutment deformation.

In addition E enters into the determination of the effect of a temperature drop on the stresses. When for example $\Delta T = 0$, (Fig. 5) and $h = 620$ ft, $n_{aa} = 1200$ psi. This does not involve E directly, only the ratio E/E_f. For $\Delta T = -20$ deg and with $c\ E = 5.5 \times 300$, $n_{aa} = 1370$ psi. If E is doubled, $n_{aa} = 1560$ psi, an increase of only 15 percent.

The computed deflections also depend upon the ratio E/E_f and in addition they depend directly on E, so that when E is doubled, the deflections are doubled, even for $\Delta T = 0$. Therefore the errors likely to be involved in the computed values of deflections are much larger than for the stresses. An example of the difficulties involved in obtaining a realistic determination of the division of load between the arches and the cantilevers may be gathered from a recent trial load analysis of Ross Dam by the Bureau of Reclamation, dated July 1944, which gave a maximum tension of 362 psi in the arches and 338 psi in the cantilevers.

There are other factors which influence the accuracy of a stress computation as outlined in this paper. So, for example, the stresses along radial planes are not linear, the temperature stresses are not uniform, as assumed, and the deformations of the abutments due to the water load on the reservoir have been disregarded and can only be roughly estimated from Vogt's formulas. This would involve an immense amount of computation of questionable validity.

On the other hand the maximum load, including earthquake shock, can be determined with considerable accuracy and excessive stresses in an arch

would be reduced by the plastic yielding of the concrete and the consequent redistribution of the stresses near the crown and the abutment. In view of this and the yield-line theory or ultimate load design, in which factors of safety are based on the ultimate strength, and which is currently before the profession,[7] it would be highly desirable to have some unreinforced concrete arches tested to destruction under normal loads and with the abutment and crown joints free to open. This would not only be interesting but good economy, for arch dams have been designed with an unreasonably high factor of safety and this has operated against the construction of arch dams in this country.

REFERENCES

1. Cain, William, *Transactions*, ASCE, V. 85, 1922, p. 233.
2. Jakobsen, B. F., *Transactions*, ASCE, V. 90, 1927, p. 475.
3. Jakobsen, B. F., *Transactions*, ASCE, V. 92, 1928, p. 1545.
4. Jakobsen, B. F., *Transactions*, ASCE, V. 90, 1927, p. 484.
5. Vogt, Fredrik, "Uber die Berechnung der Fundamentaldeformation," Norsk Videnskaps-Akademi, Oslo, 1925.
6. Vogt, Frederik, *Transactions*, ASCE, V. 90, 1927, p. 554.

7a. Hognestad, Eivind, "Fundamental Concepts in Ultimate Load Design of Reinforced Concrete Members," ACI JOURNAL, June 1952, *Proc.* V. 48, pp. 809-832.

b. Hognestad, Eivind, "Yield-Line Theory for the Ultimate Flexural Strength of Reinforced Concrete Slabs," ACI JOURNAL, Mar. 1953, *Proc.* V. 49, pp. 637-656.

c. Corning, Leo H., "Introduction to Ultimate Load Design," ACI JOURNAL, June 1952, *Proc.* V. 48, pp. 797-800.

d. Anderson, Boyd G., "Why Design by Ultimate Strength Theories?" ACI JOURNAL, June 1952, *Proc.* V. 48, pp. 801-808.

e. Siess, C. P., "Review of Research on Ultimate Strength of Reinforced Concrete Members," ACI JOURNAL, June 1952, *Proc.* V. 48, pp. 833-864.

f. Reese, R. C., "Practical Design at Ultimate Loads," ACI JOURNAL, June 1952, *Proc.* V. 48, pp. 865-880.

g. Lin, T. Y., "Load Factors in Ultimate Design of Reinforced Concrete," ACI JOURNAL, June 1952, *Proc.* V. 48, pp. 881-900.

APPENDIX

Crown segment

$u - c = 1 - (1 - R)\cos\phi - R\,z/r_c = A + B\cos\phi$ in which $A = 1 - R\,z/r_c$ and $B = -(1 - R)$ so that $(A - B)/(A + B) > 0$.

I_{cA}: the integral of $d\phi/(A + B\cos\phi)$ is given by Eq. (4), p. 20, of William F. Osgood's *Advanced Calculus* (1925). It is also given by B. O. Peirce, second revised edition, No. 300, but Osgood warns that some of the four solutions given are treacherous.

I_{cA}: the integral of $d\phi/(A + B\cos\phi)^2$ is given by Peirce (No. 308).

I_{cN}: the integral of $\cos\phi\ d\phi/(A + B\cos\phi)$; put $x = A + B\cos\phi$ so that $\cos\phi = (x - A)/B$, then

$$\frac{\cos\phi}{A + B\cos\phi} = \frac{x - A}{B} - \frac{1}{B}\left(1 - \frac{A}{A + B\cos\phi}\right)$$

I_{cM}: the integral of

$$\frac{\cos\phi}{(A + B\cos\phi)^2} = \frac{1}{B}\left(\frac{1}{x} - \frac{A}{x^2}\right)$$

I_{cS}: the integral of $\dfrac{\sin^2\phi}{A + B\cos\phi} = \dfrac{1 - \cos^2\phi}{A + B\cos\phi} = \dfrac{1 - (x - A)^2/B^2}{x}$

$$= \frac{1}{B^2}\left(\frac{B^2 - A^2}{A + B\cos\phi} + A - B\cos\phi\right)$$

Abutment

$a - u = (a - 1) + (1 - R)\cos\phi = (Rz/r_i - 1) + (1 - R)\cos\phi = C + D\cos\phi$ in which $C = (Rz/r_i - 1)$ and $D = 1 - R$ so that $(C + D)/(C - D) < 0$.

I_{ai}: the integral of $d\phi/(A + B\cos\phi)$ is given by Eq. (6) of Osgood's *Advanced Calculus* (1925), p. 20.

I_{ais}: this integral is given by Peirce (No. 308).

I_{aN}, I_{aM} and I_{aS} are obtained as I_{cN}, I_{cM} and I_{cS}, respectively.

Title No. 51-37

Prefabricated Factory Construction in Denmark*

By R. A. LARSEN† and VAGN USSING‡

SYNOPSIS

Development in methods of prestressing and prefabrication of concrete has been rapid in Denmark. The design, production, and erection of 400,000 sq ft of factory buildings are described and total erection time and cost are discussed.

INTRODUCTION

Postwar conditions in Denmark have forced Danish contractors to employ new methods of construction. The huge demand for new construction created severe shortages in skilled labor, and consequently new methods were introduced creating a more favorable ratio of demand between skilled and unskilled labor. Development in Denmark, as elsewhere, has gone in the direction of increased industrialization of the construction business. Prefabrication to the largest extent possible and employment of the newest methods such as prestressing and steam curing has resulted.

This paper describes one of the biggest factory construction jobs in Scandinavia, undertaken almost entirely with modern methods. In the spring of 1951 Northern Cable and Wireworks, Inc., decided to enlarge its plants in Copenhagen by adding a completely new division. All phases of the construction job, including the design, were handled by Larsen & Nielsen Constructor, Inc., Copenhagen.

LAY-OUT AND DESIGN

The new division was planned to comprise 400,000 sq ft of one-story buildings situated on 125 acres of farmland. The owner agreed to a high degree of standardization. Accordingly all factory areas were designed with a column spacing of 54 ft x 29.5 ft, the only variable being the height of the building. All floor areas were required to have 25 percent skylight. Several buildings were to be equipped with 10 to 15-ton overhead cranes. The exterior walls and the design in general were required to allow for easy and economical expansion of the buildings.

DESIGN DETAILS

Fig. 1 shows a typical section. The free height is here 28 ft to the main truss. From the very beginning complete prefabrication was considered in

*Received by the Institute Aug. 20, 1954. Title No. 51-37 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Aug. 1, 1955. Address 18263 W. McNichols Road, Detroit 19, Mich.

†Member American Concrete Institute, Director, Larsen & Nielsen Constructor A/S, Copenhagen, Denmark.
‡Assistant Director, Larsen & Nielsen Constructor A/S, Copenhagen, Denmark.

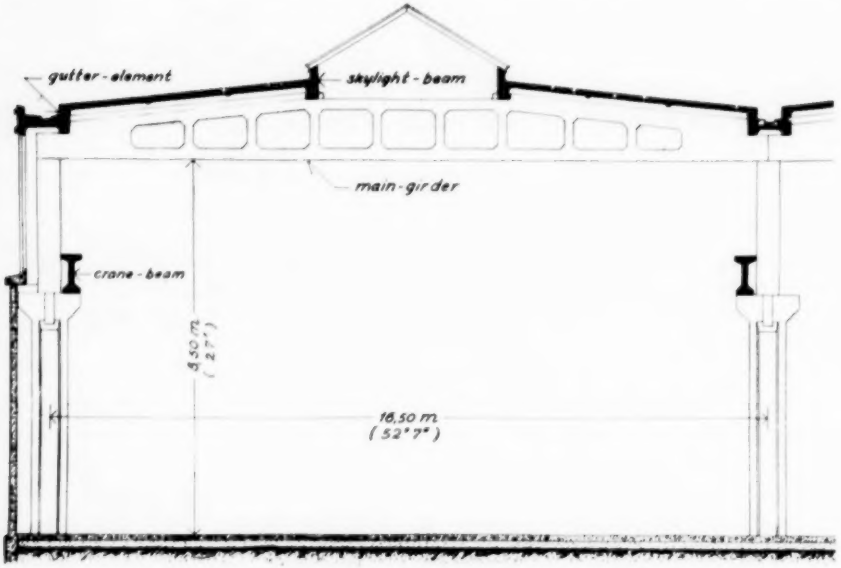


Fig. 1—Typical section of outer bay

the design. Consequently all horizontal forces were carried by fixing the columns at the bottom. The joint between the main truss and the columns were considered as having no stiffness. Thus the main trusses may be considered as only distributing horizontal forces in the respect that the trusses cause identical deflections in the columns, which really can be considered as cantilever beams fixed in the footings. This means that all lateral forces are distributed to the columns according to their relative stiffness. The joints over the columns were cast only after nearly all dead load was on the roof, and no cracks have been observed from any subsequent rotation of the joints despite the lack of reinforcement.

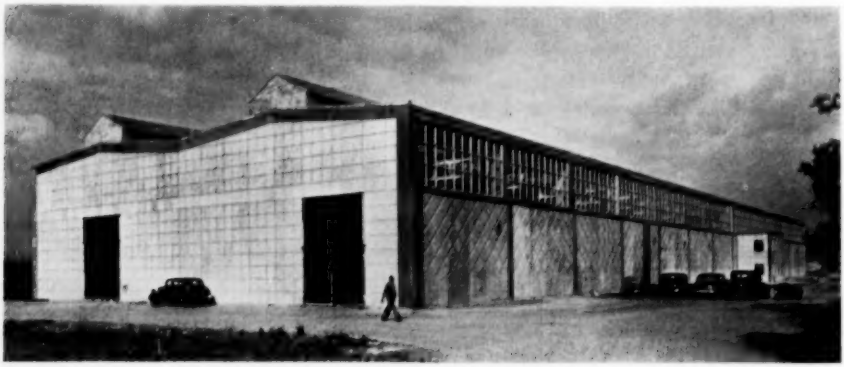


Fig. 2—Exterior of finished building

Fig. 3—General view of beam casting yard

The shape of the columns accommodates pipes and electrical installations and give maximum effective area for the cranes. Due to the crane brackets, the walls were placed 1 ft outside the columns thus permitting pipes and ducts to pass straight along the walls.

The slope of the roof was taken as 1: 10 and the main truss was designed to utilize the space thus created. To keep the weight low a prestressed Vierendeel truss was used.

Due to the required skylight four longitudinal girders were introduced, two of which support the skylight. These longitudinal girders are reinforced with ordinary reinforcement for the dead load of the girder itself and post-tensioned by the Freyssinet method after erection thus making the girders continuous for any further dead loads and all live loads.

The same principle was applied to the crane beams. As the crane capacities varied, one I-shape was standardized and only the number of prestressing cables was varied according to the load.

The longitudinal prestress creates a good continuity in the structure, but length of cables was limited by friction and the size of the Freyssinet jacks available. Thus every 250 ft a double column spaced 8 ft center to center was necessary to allow for the prestressing operation and thermal movement of the structure.

The design of all prestressed members were based on the ultimate resistance in bending being larger than 1.5 times the dead load and 2.5 times the live load. It was assumed that strain distribution was linear over the section. For the evaluation of ultimate bending resistance of the prestressed beams, the ultimate strain in the concrete was taken as 0.005 and the corresponding constant average concrete stress as 4300 psi assuming that the ultimate concrete strength is the average of 50 cylinders reduced by two times the average deviation from the mean.

The prestressing steel had a guaranteed ultimate strength of 240,000 psi on 5-mm (0.2 in.) wires. The initial prestress was 156,000 psi and subsequent loss due to creep and shrinkage was set at 21,000 psi.

The ultimate strains for ordinary steel, if any, were considered larger than that of the prestressed cables, thus permitting the mild steel to be considered as carrying its part of the ultimate stresses.

The concrete was designed to be at least 7000 psi (6 x 12-in. cylinders) and the maximum stress permitted at the initial stressing or any subsequently applied actual load was 2,900 psi.

Between the longitudinal girders 1¾-in. thick slabs with ribs 6 ft center to center were used. Reinforcement in the ribs was ¾-in. corrugated bars. The roof slabs were designed for all dead loads plus 16 psf snow or 330-lb concentrated load whichever was the worst condition. Again ultimate theory was used. The roof slabs were insulated after erection with 2 in. of woodcrete allowing for ventilation through two layers of bitumen felt.

The exterior walls were all of the sandwich type, the insulation being 4½ in. of "Leca-concrete." This concrete is made with expanded clay aggregate, and the unit weight of the fresh mix is around 50 lb per cu ft. The

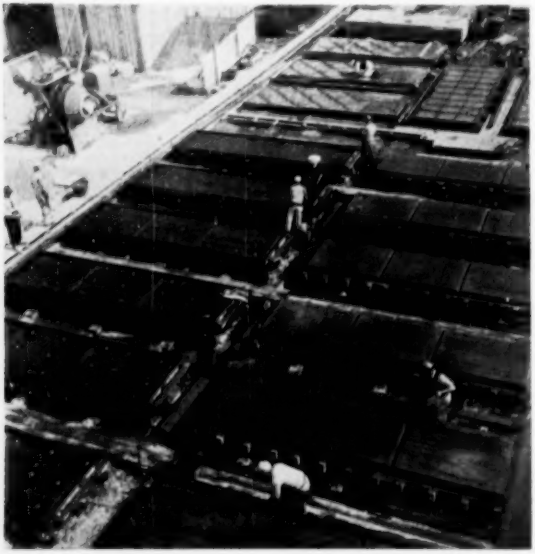


Fig. 4—Slab casting area

Fig. 5—Casting of columns

sandwich walls are made up of 1¾ in. of reinforced concrete, 4½ in. of Leca-concrete, and 1¼ in. of reinforced concrete on the inside. Reinforced concrete ties between the outer and inner layers are provided. As shown in Fig. 1 the window sill serves as a wind brace between the columns. The concrete window frames are factory made.

Skylights were designed as three-hinged frames of prestressed mullions 2½ ft center to center. The cross section of the mullions is 2¼ x 3½ in., the reinforcement is prestressed notched wires.

A major object of the design was to reduce maintenance and create the highest degree of fire resistance. For architectural purposes certain patterns were designed as decorative elements for the wall panels. High re-use of forms made rather elaborate patterns economically possible, the texture of the pattern changing between the polished smoothness of steel plates and rough impressions of wood in the forms (Fig. 2).

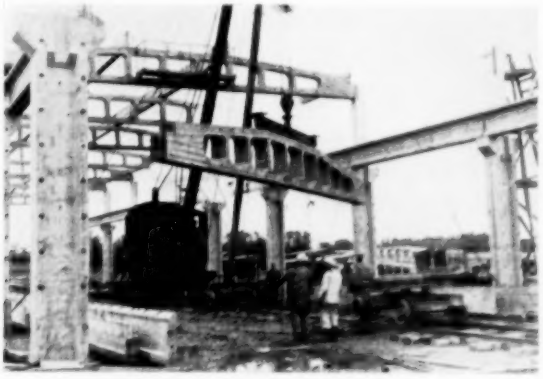


Fig. 6—Erection of main truss and roof

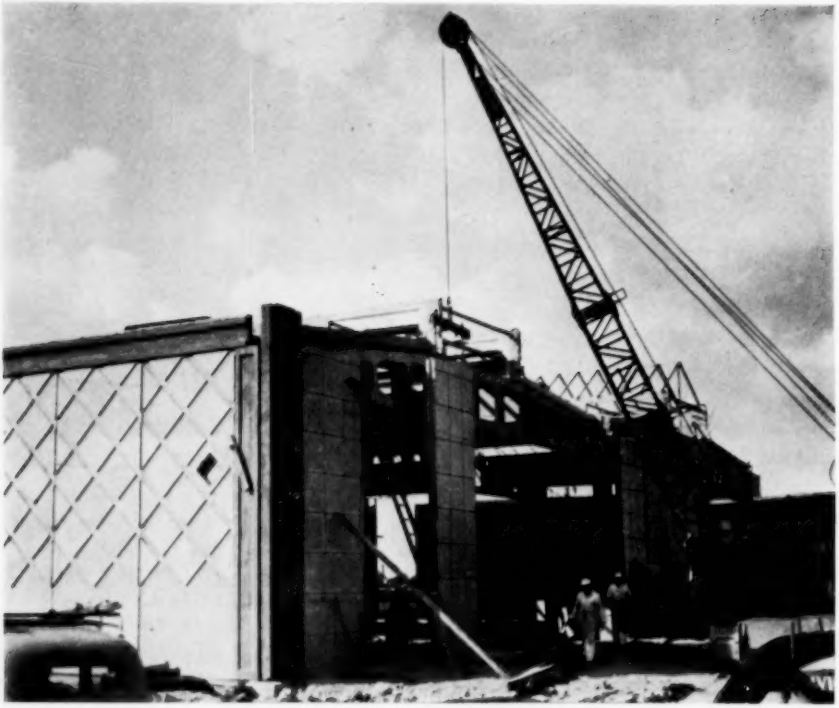


Fig. 7—Erection of wall panels

PRODUCTION AND ERECTION

All components were produced in open air casting yards. Roof and wall panels were made in one 350-ft yard served by a 40 ft wide crane-frame with electric hoist (Fig. 3). Beams and trusses as well as the decorative dark components of the exterior walls were produced in a similar yard parallel to the first (Fig. 4). The casting yards were designed to produce around 1500 sq ft (maximum) per 8-hr day corresponding to one bay of the roof structure.

The mixers were one Eirich-type and three Kaiser-type mixers. All concrete was placed by vibration, all slabs and beams being externally vibrated. Duct-tubes were used to form the holes for the prestressing cables. All parts were carefully marked thus making identification of the corresponding test specimens easy.

Concrete was tested in a locally erected laboratory equipped with Norwegian equipment. Test specimens were 4 x 8-in. cylinders cast vertically, compacted by a "fall handle" method, and stored horizontally in damp sand.

All components were cast well ahead of erection, thus permitting initial shrinkage during storage in the casting yard. This demanded careful plan-

Fig. 8—Interior of one 40,000 sq ft building

ning of foundation work and erection to prevent the yard from being "soaked in."

Columns were cast in place in steel forms placed around the column steel which had been assembled, erected, and cast into the footing (Fig. 5).

Erection of precast elements was by steam cranes running on rails. One crane worked on the roof structure and one on the wall panels. Erection of the roof structure reached 9000 sq ft per week, as the workers adapted themselves to the new methods. Thus erection and all other operations were geared to the production of the casting yard permitting accurate time schedules to be worked out (Fig. 6 and 7).

Fig. 9—Interior of 160,000 sq ft building during completion of floor

OVER-ALL ERECTION TIME AND COST

Excavations (110,000 cu yd) were started Apr. 11, 1951, concreting of the first foundations were around May 15 and by July 1 a small casting yard was in operation. By January 1, 1952, 40,000 sq ft and a big power station had been completed. By July, 1952, 40,000 sq ft additional were completed and by July, 1953, in all 240,000 sq ft, this being increased to 370,000 sq ft by October 1 leaving only 40,000 to be finished before January 1, 1954. Time schedules have been met for all buildings except the first ones, where minor difficulties in training the labor force and selecting proper equipment created small deviations. Fig. 8 shows the interior of the 40,000 sq ft factory and Fig. 9 shows the columns in 160,000 sq ft of factory under one single roof.

Average cost of the factory buildings, as shown in Fig. 1, including roofing, skylights, drainage, crane beams, walls and windows, foundations (spread footing), and a 6-in. steel-troweled floor was $3.00 per sq ft or $0.10 per cu ft.* The ribbed roof panels were produced and erected for around $0.35 to $0.40 per sq ft.

*Cost includes design and supervision. Conversion of costs to U. S. currency based on $1.00 = 6.92 Danish Kroner. Wage rates for unskilled workers $0.75 per hr, skilled $0.85, and for civil engineers $1.00 per hr. Workers wages were about 25 percent of total cost. Materials: cement—$3.20 per bbl, sand and gravel—$1.50 per ton, mild steel—$6.50 per 100 lb, and prestressing steel—$13.00 per 100 lb.

Title No. 51-38

Strength Variations in Ready-Mixed Concrete*

By A. E. CUMMINGS†

SYNOPSIS

Presents the results of a statistical analysis of the variations in crushing strength of concrete test cylinders made of commercial ready-mixed concrete delivered to a construction job. The investigation included approximately 1000 cu yd of concrete. Test cylinders were made from every batch delivered to the job for a period of several weeks and the 7-day and 28-day strengths were determined by crushing tests in the laboratory. The test results were then subjected to mathematical analysis by statistical methods to determine the coefficient of variation of the crushing strengths.

BASIS OF STUDY

The quality control of concrete, either ready-mixed or job-mixed, is a problem of considerable technical and economic importance. The Danish National Institute of Building Research has been studying this subject for a number of years and the results of their investigations have recently been published.‡ Among other things, the Danish report describes a procedure for studying variations in the quality of concrete by means of statistical methods.

Variations in the quality of concrete produced in batches may be placed in two general categories:

(a) Variations within an individual batch which could be caused by imperfect mixing, by segregation of aggregates during discharge operations, or by variations in making, curing, and testing cylinders.

(b) Variations from batch to batch which could be caused by imperfect control of the mix proportions or by variations in the quality of cement and aggregate.

TEST PROCEDURE

On a recent construction project in the middle west it was possible to secure sufficient test data to justify a statistical analysis of the variations in the crushing strength of approximately 1000 cu yd of ready-mixed concrete delivered in 208 separate batches. The transit-mix trucks were loaded at the batch plant. Water was put in the drum first, then the aggregates were added, and the cement was put on top of the aggregates. No mixing was done while the truck was enroute from the batch plant to the job. When

*Received by the Institute Jan. 4, 1954. Title No. 51-38 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints are available at 35 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Aug. 1, 1955. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Director of Research, Raymond Concrete Pile Co., New York, N. Y.

‡Plum, Niels M., "Quality Control of Concrete—Its Rational Basis and Economic Aspects," Paper 5879, *Proceedings*, Institution of Civil Engineers, London, May 1953, p. 311.

the truck reached the job site, water was added if required and the mixer drum was rotated for the specified length of time.

Test cylinders were made from every batch of concrete delivered to the job over a period of several weeks. One cylinder was taken from each batch for a 7-day test and another cylinder for a 28-day test. The cylinders were cured in air on the job over night and then taken to the laboratory the day after casting. At the laboratory they were cured in tanks of water until the day before testing when they were removed from the water, capped, and allowed to cure in air in the laboratory until they were put in the testing machine.

The sampling and testing procedure used on this project did not provide the data that would be required to study the variations within the individual batches. For such an investigation it would be necessary to test a large number of cylinders from a single batch. The statistical analysis of the test data is therefore made to determine the variations from batch to batch. This constitutes a check on the mix control at the batching plant and on the uniformity of the quality of the cement and aggregates. In addition, the test results can be used to show the variations in quality from day to day.

The concrete mix was designed by the Pittsburgh Testing Laboratory for a compressive strength of 5200 psi at 28 days. The cement was ASTM Type I and the aggregates were sand and gravel with 1-in. maximum gravel. The cement factor was 7 sacks per cu yd and the water-cement ratio 5 gal. per sack. The proportions per cubic yard by weight were 658 lb of cement, 1160 lb of sand, and 2040 lb of gravel. The volumetric proportions were 1 : 1.44 : 2.62.

TEST RESULTS

Table 1 contains results of the tests on the 7-day cylinders. In column (1) are the dates on which the concrete was cast and column (2) shows the crushing strengths in psi of each of the 208 cylinders tested. The sum of all the crushing strengths in column (2) is divided by the number of tests to determine the arithmetic mean $\bar{X}$, which is 4017 psi. Column (3) shows the differences between the arithmetic mean and the actual strength of each individual cylinder. Of the 208 cylinders tested, 117 were above the mean and 91 were below. Column (4) shows the squares of the differences in column (3). The sum of the squares in column (4) is used to calculate the standard deviation, σ, from the following formula:

$$\sigma = \sqrt{\frac{\sum_{1}^{N}(X - \bar{X})^2}{N}} \qquad (1)$$

The standard deviation amounts to 464 psi. The coefficient of variation V is a measure of the relative dispersion of the data and is expressed as a percentage by the formula:

$$V = \frac{\sigma}{\bar{X}} \qquad (2)$$

TABLE 1—7-DAY CYLINDER STRENGTHS

(1) Date	(2) X, psi	(3) $(X - \bar{X})$	(4) $(X - \bar{X})^2$
May 25	4346	+ 329	108,241
	3852	− 165	27,225
	4240	+ 223	49,729
	3109	− 908	824,464
	3887	− 130	16,900
	3852	− 165	27,225
	3263	− 754	568,516
	3781	− 236	55,696
May 26	4346	+ 329	108,241
	4064	+ 47	2,209
	3498	− 519	269,361
	4170	+ 153	23,409
	4629	+ 612	374,544
	4452	+ 435	189,225
	4170	+ 153	23,409
	4558	+ 541	292,681
	3640	− 377	142,129
	4488	+ 471	221,841
	4205	+ 188	35,344
	4806	+ 789	622,521
	4099	+ 82	6,724
	4134	+ 117	13,689
	2650	−1367	1,868,689
	3427	− 590	348,100
May 27	3852	− 165	27,225
	3993	− 24	576
	4346	+ 329	108,241
	4028	+ 11	121
	4594	+ 577	332,929
	4099	+ 82	6,724
	4240	+ 223	49,729
	3852	− 165	27,225
	4346	+ 329	108,241
	4134	+ 117	13,689
	4064	+ 47	2,209
	4028	+ 11	121
	4064	+ 47	2,209
	4841	+ 824	678,976
	4735	+ 718	515,524
	4064	+ 47	2,209
	4558	+ 541	292,681
	3887	− 130	16,900
	4488	+ 471	221,841
	4664	+ 647	418,609
	4099	+ 82	6,724
	4099	+ 82	6,724
May 28	4170	+ 153	23,409
	4028	+ 11	121
	4240	+ 223	49,729
	4064	+ 47	2,209
	4099	+ 82	6,724
	4240	+ 223	49,729
	4558	+ 541	292,681
	4523	+ 506	256,036
	4558	+ 541	292,681
	4594	+ 577	332,929
	4417	+ 400	160,000
	4700	+ 683	466,489
	4028	+ 11	121
	4311	+ 294	86,436
	4134	+ 117	13,689
	4311	+ 294	86,436
	4488	+ 471	221,841
	4346	+ 329	108,241
	4558	+ 541	292,681
	4205	+ 188	35,344
	3710	− 307	94,249
	4276	+ 259	67,081
May 29	4488	+ 471	221,841
	3922	− 95	9,025
	3357	− 660	435,600
	3286	− 731	534,361
	3604	− 413	170,569
	3781	− 236	55,696
May 29	3816	− 201	40,401
	3534	− 483	233,289
	4028	+ 11	121
	4064	+ 47	2,209
	3498	− 519	269,361
	3956	− 61	3,721
	4064	+ 47	2,209
	4028	+ 11	121
	4346	+ 329	108,241
	4064	+ 47	2,209
	4629	+ 612	374,544
	4346	+ 329	108,241
	4523	+ 506	256,036
	4311	+ 294	86,436
	4134	+ 117	13,689
	3852	− 165	27,225
June 1	5230	+1213	1,471,369
	2430	−1587	2,518,569
	4876	+ 859	737,881
	4311	+ 294	86,436
	4382	+ 365	133,225
	4594	+ 577	332,929
	3781	− 236	55,696
	2720	−1297	1,682,209
	3887	− 130	16,900
	3781	− 236	55,696
	3710	− 307	94,249
	3463	− 554	306,916
	3746	− 271	73,441
June 3	4346	+ 329	108,241
	4382	+ 365	133,225
	4240	+ 223	49,729
	4700	+ 683	466,489
	4346	+ 329	108,241
	4523	+ 506	256,036
	4064	+ 647	418,609
	4841	+ 824	678,976
	4523	+ 506	256,036
	4134	+ 117	13,689
	4488	+ 471	221,841
	3321	− 696	484,416
	3922	− 95	9,025
	3993	− 24	576
	3816	− 201	40,401
	3922	− 95	9,025
	3180	− 837	700,569
June 4	4240	+ 223	49,729
	3746	− 271	73,441
	4099	+ 82	6,724
	4134	+ 117	13,689
	4664	+ 647	418,609
	4311	+ 294	86,436
	3956	− 61	3,721
	4211	+ 194	37,636
	4488	+ 471	221,841
	4134	+ 117	13,689
	4523	+ 506	256,036
	3852	− 165	27,225
	4346	+ 329	108,241
	4558	+ 541	292,681
	4170	+ 153	23,409
	4060	+ 43	1,849
	4188	+ 471	221,841
June 9	4488	+ 471	221,841
	4806	+ 789	622,521
	3710	− 307	94,249
	4064	+ 47	2,209
	4099	+ 82	6,724
	4488	+ 471	221,841
	4134	+ 117	13,689
	3956	− 61	3,721
	3993	− 24	576
	4134	+ 117	13,689
	4205	+ 188	35,344
June 9	4452	+ 435	189,225
	3781	− 236	55,696
	4594	+ 577	332,929
	4356	+ 339	114,921
	3180	− 837	700,569
June 10	3569	− 448	200,704
	3039	− 978	956,484
	3922	− 95	9,025
	3887	− 130	16,900
	3887	− 130	16,900
	4064	+ 47	2,209
	3746	− 271	73,441
	3640	− 377	142,129
	4488	+ 471	221,841
	4488	+ 471	221,841
	4064	+ 47	2,209
	3427	− 590	348,100
	3887	− 130	16,900
	3956	− 61	3,721
	3816	− 201	40,401
	4240	+ 223	49,729
	3534	− 483	233,289
	3746	− 271	73,441
	3427	− 590	348,100
	3852	− 165	27,225
	3781	− 236	55,696
	3498	− 519	269,361
	4134	+ 117	13,689
June 11	3710	− 307	94,249
	3074	− 943	889,249
	3321	− 696	484,416
	2862	−1155	1,334,025
	3109	− 908	824,464
	3321	− 696	484,416
	3534	− 483	233,289
	3956	− 61	3,721
	3746	− 271	73,441
	3781	− 236	55,696
	3710	− 307	94,249
	3392	− 625	390,625
	3887	− 130	16,900
	3816	− 201	40,401
	3993	− 24	576
	3286	− 731	534,361
	3604	− 413	170,569
	3604	− 413	170,569
	2827	−1190	1,416,100
	3427	− 590	348,100
	3463	− 554	306,916
	3321	− 696	484,416
	3215	− 802	643,204
June 12	3392	− 625	390,625
	3852	− 165	27,225
	3640	− 377	142,129
	4064	+ 47	2,209
	4276	+ 259	67,081
	3463	− 554	306,916
	4523	+ 506	256,036
	4382	+ 365	133,225
	4346	+ 329	108,241

$\Sigma X = 835,591$ $\Sigma (X - \bar{X})^2 = 44,831,695$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{835,591}{208} = 4017.264423$$

Use 4017 psi

For the 7-day cylinders tested on this project the coefficient of variation was 11.55 percent.

Fig. 1 is a graphic representation of the results of the tests on the 7-day cylinders in the form of a histogram. The test results are grouped in class intervals of 200 psi ranging from 2400 to 5400 psi. The compressive strengths are plotted as abscissas and the ordinates represent the number of cylinders in each class interval. The stepped diagram is a plot of the actual test data. The dotted curve is a continuous Gaussian frequency distribution determined from the formula:

$$Y_x = \frac{Ni}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \quad (3)$$

in which

Y_x = ordinate at distance x from arithmetic mean
N = number of test results = 208
i = class interval = 200 psi
σ = standard deviation = 464 psi
x = any selected deviation from the arithmetic mean
π = a constant = 3.1416
e = Naperian log base = 2.71828

The maximum ordinate of the continuous distribution is located at the arithmetic mean where $x = 0$. It is calculated from the formula:

$$Y_{max} = \frac{N\,i}{\sigma\sqrt{2\pi}} \quad (4)$$

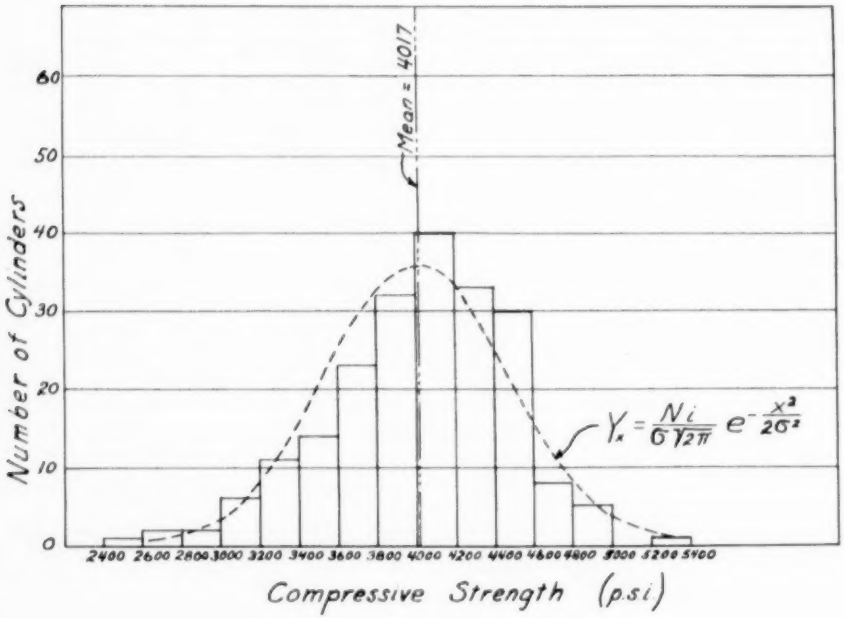


Fig. 1—Histogram for 7-day cylinders

As can be seen from Fig. 1, the actual test results represented by the stepped diagram correspond quite well with the theoretical frequency curve. There is a slight skewness but 50 percent of the cylinder strengths lie between 3705 and 4329 psi. This region is bounded by the area under the curve and the two ordinates plotted at $\pm 0.6745\sigma$ each way from the arithmetic mean. The median, which divides the distribution so that an equal number of test results are on either side of it, is approximately 4047 psi.

Table 2 contains test data for the 28-day cylinders arranged in the same manner as in Table 1. The arithmetic mean for the 28-day cylinders is 5249 psi. In the column of differences there are 105 values above the mean and 103 values below it. The standard deviation calculated by Eq. (1) is 490 psi and the coefficient of variation by Eq. (2) is 9.34 percent.

Fig. 2 shows the frequency distribution for the 28-day cylinders. The stepped diagram representing the actual test data is based on class intervals of 200 psi as in Fig. 1. The fitted theoretical frequency curve is calculated from Eq. (3) and (4) with $N = 208$; $i = 200$ psi, and $\sigma = 490$. There is some skewness in the actual test results but 50 percent of the cylinder strengths lie between 4919 and 5579 psi. The median is approximately 5209 psi.

On the basis of many investigations in the field and in the laboratory, the Danish National Institute of Building Research has established a classification system in which the quality of the workmanship is related to the

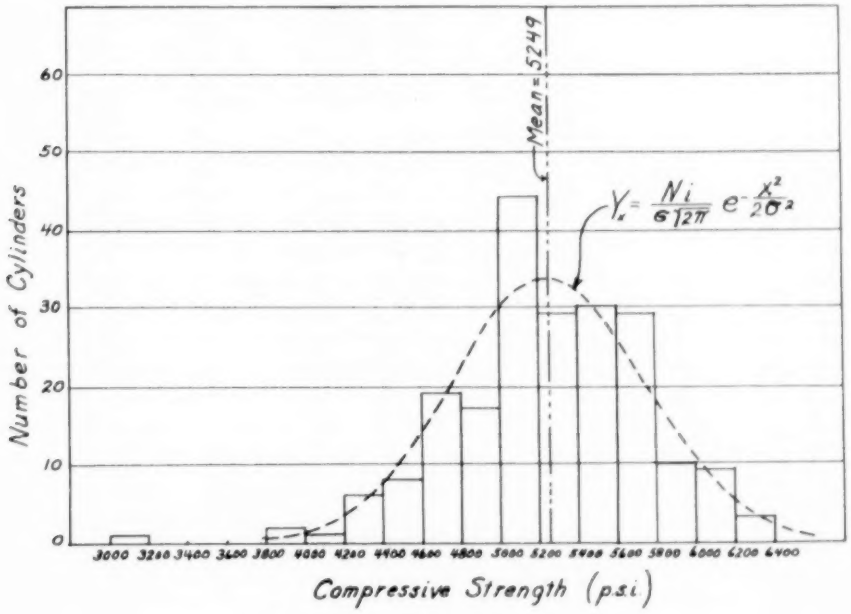


Fig. 2—Histogram for 28-day cylinders

TABLE 2—28-DAY CYLINDER STRENGTHS

(1) Date	(2) X, psi	(3) $(X - \bar{X})$	(4) $(X - \bar{X})^2$
May 25	5018	− 231	53,361
	4523	− 726	577,076
	5442	+ 193	37,249
	5052	− 197	38,809
	5159	− 90	8,100
	5618	+ 369	136,161
	4735	− 514	264,196
	4806	− 443	196,249
May 26	5583	+ 334	111,556
	5265	+ 16	256
	5689	+ 440	193,600
	5194	− 55	3,025
	5512	+ 263	69,169
	5124	− 125	15,625
	5230	− 19	361
	5760	+ 511	261,121
	5159	− 90	8,100
	5371	+ 122	14,884
	6007	+ 758	574,564
	6113	+ 864	746,496
	5795	+ 546	298,116
	5866	+ 617	380,689
	5159	− 90	8,100
	5724	+ 475	225,625
May 27	5618	+ 369	136,161
	5512	+ 263	69,169
	5760	+ 511	261,121
	5194	− 55	3,025
	6219	+ 970	940,900
	5724	+ 475	225,625
	5371	+ 122	14,884
	5124	− 125	15,625
	5548	+ 299	89,401
	5724	+ 475	225,625
	5512	+ 263	69,169
	5371	+ 122	14,884
	5442	+ 193	37,249
	5654	+ 405	164,025
	6113	+ 864	746,496
	5583	+ 334	111,556
	5618	+ 369	136,161
	5052	− 197	38,809
	6007	+ 758	574,564
	5548	+ 299	89,401
	6254	+ 1005	1,010,025
	6254	+ 1005	1,010,025
May 28	5230	− 19	361
	5088	− 161	25,921
	5371	+ 122	14,884
	5124	− 125	15,625
	5265	+ 16	256
	5618	+ 369	136,161
	5689	+ 440	193,600
	5583	+ 334	111,556
	5194	− 55	3,025
	5795	+ 546	298,116
	5972	+ 723	522,729
	5618	+ 369	136,161
	5548	+ 299	89,401
	5265	+ 16	256
	5583	+ 334	111,556
	5477	+ 228	51,984
	5936	+ 687	471,969
	5124	− 125	15,625
	4629	− 620	384,400
	5583	+ 334	111,556
	5194	− 55	3,025
	5830	+ 581	337,561
May 29	5477	+ 228	51,984
	5052	− 197	38,809
	4523	− 726	527,076
	4700	− 549	301,401
	4664	− 585	342,225
	5088	− 161	25,921
May 29	4770	− 479	229,441
	4806	− 443	196,249
	4947	− 302	91,204
	5336	+ 87	7,569
	4558	− 691	477,481
	5194	− 55	3,025
	5724	+ 475	225,625
	5794	+ 545	297,025
	5724	+ 475	225,625
	4876	− 373	139,129
	5371	+ 122	14,884
	5830	+ 581	337,561
	5830	+ 581	337,561
	6113	+ 864	746,496
	6007	+ 758	574,564
	5300	+ 51	2,601
June 1	5901	+ 652	425,104
	5936	+ 687	471,969
	5795	+ 546	298,116
	6007	+ 758	574,564
	5724	+ 475	225,625
	5583	+ 334	111,556
	4912	− 337	113,569
	4558	− 691	477,481
	5300	+ 51	2,601
	4594	− 655	429,025
	5654	+ 405	164,025
	4311	− 938	879,844
	5194	− 55	3,025
June 3	5230	− 19	361
	5124	− 125	15,625
	5230	− 19	361
	5300	+ 51	2,601
	5265	+ 16	256
	4240	− 1009	1,018,081
	5654	+ 405	164,025
	5972	+ 723	522,729
	5583	+ 334	111,556
	5088	− 161	25,921
	5477	+ 228	51,984
	4982	− 267	71,289
	4735	− 514	264,196
	5018	− 231	53,361
	4841	− 408	166,464
	4735	− 514	264,196
	3003	− 2246	5,044,516
June 4	5088	− 161	25,921
	4700	− 549	301,401
	4947	− 302	91,204
	4912	− 337	113,569
	5194	− 55	3,025
	5124	− 125	15,625
	5265	+ 16	256
	5124	− 125	15,625
	5336	+ 87	7,569
	5052	− 197	38,809
	5230	− 19	361
	5442	+ 193	37,249
	5512	+ 263	69,169
	5618	+ 369	136,161
	5300	+ 51	2,601
	5194	− 55	3,025
	5265	+ 16	256
June 9	5760	+ 511	261,121
	5901	+ 652	425,104
	5371	+ 122	14,884
	5724	+ 475	225,625
	5795	+ 546	298,116
	5548	+ 299	89,401
	5018	− 231	53,361
	5159	− 90	8,100
	5406	+ 157	24,649
	5371	+ 122	14,884
	5265	+ 16	256
June 9	5795	+ 546	298,116
	5265	+ 16	256
	5689	+ 440	193,600
	5088	− 161	25,921
	5088	− 161	25,921
June 10	4735	− 514	264,196
	4558	− 691	477,481
	5124	− 125	15,625
	5018	− 231	53,361
	5088	− 161	25,921
	5371	+ 122	14,884
	4876	− 373	154,013
	4735	− 514	264,196
	6007	+ 758	574,564
	6042	+ 793	628,849
	4841	− 408	166,464
	5052	− 197	38,809
	4488	− 761	579,121
	5689	+ 440	193,600
	4912	− 337	113,569
	5159	− 90	8,100
	5194	− 55	3,025
	5124	− 125	15,625
	4735	− 514	264,196
	4760	− 549	301,401
	4735	− 514	264,196
	4735	− 514	264,196
	5371	+ 122	14,884
June 11	4240	− 1009	1,018,081
	4028	− 1221	1,490,841
	4346	− 903	815,409
	4276	− 973	946,729
	3887	− 1362	1,855,044
	4664	− 585	342,225
	4346	− 903	815,409
	5548	+ 299	89,401
	4700	− 549	301,401
	5406	+ 157	24,649
	5194	− 55	3,025
	4806	− 443	196,249
	5442	+ 193	37,249
	4735	− 514	264,196
	5194	− 55	3,025
	4700	− 549	301,401
	4770	− 479	229,441
	5088	− 161	25,921
	3887	− 1362	1,855,044
	5124	− 125	15,625
	4982	− 267	71,289
	4841	− 408	166,464
	4558	− 691	477,481
June 12	4841	− 408	166,464
	5583	+ 334	111,556
	5371	+ 122	14,884
	5548	+ 299	89,401
	5442	+ 193	37,249
	4947	− 302	91,204
	5477	+ 228	51,984
	5442	+ 193	37,249
	5052	− 197	38,809

$\Sigma X = 1{,}091{,}753$

$\Sigma (X - \bar{X})^2 = 49{,}895{,}414$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{1{,}091{,}753}{208} = 5248.8125$$

Use 5249 psi

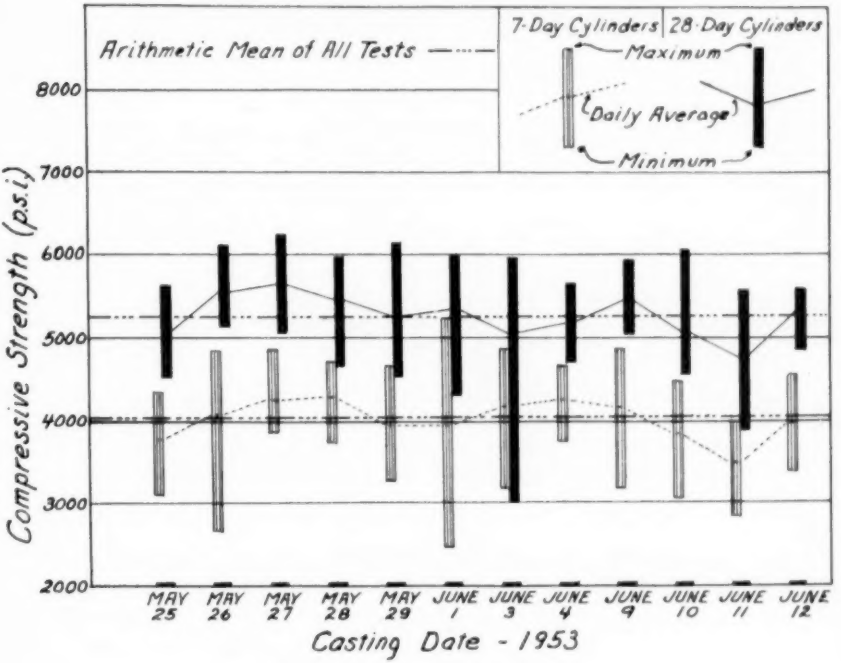


Fig. 3—Daily strength variations

coefficient of variation. This classification system as given in the Danish report is shown in Table 3. With a coefficient of variation of 11.55 percent for the 7-day cylinders and 9.34 percent for the 28-day cylinders all of the results on this project would be classified as excellent. The results on the 28-day cylinders approach laboratory precision.

Fig. 3 is a graphic representation of the day to day variations in the strength of the concrete. The length of one of the vertical bars is a measure of the uniformity of the mix—the shorter the bar the more uniform the mix. Overlapping of the bars for a given date indicates that the best of the 7-day cylinders was stronger than the poorest of the 28-day cylinders for that day. The extreme lengths of the bars for June 1 and June 3 indicate large variations in the strength of the concrete placed on those two days. However, examination of the data in Table 1 indicates that there were several "freaks" among the 7-day cylinders cast on June 1. The first cylinder for that day was 1213 psi above the arithmetic mean and the second for the day was 1587 psi below the arithmetic mean. From the data in Table 2 it can be seen that the last of the 28-day cylinders cast on June 3 was also a freak. It was 2246 psi below the arithmetic mean. These freaks account for the extreme lengths of the bars for June 1 and June 3 in Fig. 3. The existence of such freaks could be due to errors in casting the cylinders, or in capping them, or even in testing them. The average strength fluctuates from day to day but the

TABLE 3—RELATION OF VARIATION IN STRENGTH TO QUALITY OF WORKMANSHIP

Coefficient of variation, percent	Quality of workmanship
5	Attainable only in well controlled laboratory work
10	Excellent—approaches laboratory precision
12	Excellent
15	Good
18	Fair
20	Fair—minus
25	Bad

curves of daily average strengths for the two sets of cylinders are practically parallel. For both sets of cylinders the daily averages fluctuate by several hundred psi above and below the arithmetic mean for the entire set.

CONCLUSION

Studies of this kind to investigate the quality variations in concrete are of great practical usefulness. This method of analysis will show whether or not the control is satisfactory and, if the variations are excessive, the method can be used to locate the cause of the trouble. The full significance of quality control of concrete—particularly its economic aspects—is discussed in the Danish report. When the engineer can be sure of getting concrete with a high degree of uniformity in quality and strength, he can design with reduced safety factors or lower the cement factor and he can expect reduced maintenance costs on structures exposed to weathering.

ACKNOWLEDGMENT

The job on which this study was made was the Jeffries Housing Project in Detroit. The test cylinders were made under the direction of the engineers of the O. W. Burke Co., general contractors for the work. The testing was done in the laboratories of the Department of Public Works of the City of Detroit. The ready-mixed concrete was furnished by the Thomas E. Currie Co. of Detroit and was used in the Raymond cast-in-place concrete piles which support the building.

The author acknowledges with thanks the assistance of G. W. Heilbrun, who collected the data in the field, and of Miss Emma Roper who assisted with the mathematical calculations and prepared the manuscript for publication. The statistical analysis is based on the method developed by the Danish National Institute of Building Research.

Balanced Design of Prestressed Concrete Beams*

By HENRY J. COWAN†

SYNOPSIS

Equations are derived for the design of prestressed sections to develop the full value of permissible stresses in concrete both at the top and bottom of the beam during the prestressing operation as well as under the action of superimposed loads. A simple solution is obtained if the maximum permissible stresses are the same during the prestressing operation and under load. The more complicated solution for differential permissible stresses, however, gives greater economy in materials. Two examples are worked out, which demonstrate that balanced design is primarily useful for the heavily loaded beam and the beam with restricted depth.

INTRODUCTION

When a concrete beam is prestressed eccentrically by a force F_o at a distance e from the centroid of the section, a negative bending moment is set up which gives rise to large compressive stresses in the concrete near the bottom face and to small compressive or tensile stresses near the top face. If the maximum permissible concrete stresses during the prestressing operation are f_{cB} in compression and f_{tP} in tension, the section must satisfy the inequalities

$$\frac{F_o}{A_c} - \frac{F_o e}{Z_t} + \frac{M_G}{Z_t} \geq -f_{tP} \quad (1)$$

and

$$\frac{F_o}{A_c} + \frac{F_o e}{Z_b} - \frac{M_G}{Z_b} \leq f_{cP} \quad (2)$$

where A_c is the cross-sectional area of the section, Z_b and Z_t are the section moduli of the bottom and top fibers, and M_G is the bending moment due to the dead load when the prestress is established.

After the superimposed load is applied, the maximum compressive stress occurs at the top of the beam, and the minimum compressive stress or maximum tensile stress occurs at the bottom of the beam. If the maximum permissible concrete stresses under the action of the superimposed load are f_{cL} in compression and f_{tL} in tension, the section must then satisfy the inequalities

$$\frac{\eta F_o}{A_c} - \frac{\eta F_o e}{Z_t} + \frac{M_G}{Z_t} + \frac{M_S}{Z_t} \leq f_{cL} \quad (3)$$

and

$$\frac{\eta F_o}{A_c} + \frac{\eta F_o e}{Z_b} - \frac{M_G}{Z_b} - \frac{M_S}{Z_b} \geq -f_{tL} \quad (4)$$

*Received by the Institute Nov. 23, 1953. Title No. 51-39 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Aug. 1, 1955. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Member American Concrete Institute, Professor of Architectural Science, University of Sydney, Sydney, Australia.

where M_S is the bending moment due to the superimposed dead and live load, and F_o is the prestressing force remaining after loss due to creep.

Every prestressed concrete section must satisfy the simultaneous inequalities (1) to (4). It is evidently possible to design a section in which the maximum permissible stresses are just reached both at the top and at the bottom of the beam during the prestressing operation as well as under the action of the superimposed load. This is termed a *balanced design*. In prestressed concrete, as in normal reinforced concrete construction, a balanced design does not necessarily produce the most economical section.

NOTATION

A_c = net area of concrete section
A_s = cross-sectional area of steel
b_b, b_t = width of bottom, top flange of concrete section
b' = width of rib of concrete section
d_b = distance of center of steel from extreme bottom fiber
e = distance of center of steel from centroid of concrete section (eccentricity)
f_c = maximum permissible compressive concrete stress
f_{cP}, f_{cL} = maximum permissible compressive concrete stress during prestressing operation, under the action of superimposed loads
f_s = maximum permissible steel stress
f_t = maximum permissible tensile concrete stress (equal to zero, if tension in the concrete is not permitted)
f_{tP}, f_{tL} = maximum permissible tensile concrete stress during prestressing operation, under the action of superimposed loads
F_o = prestressing force when concrete is stressed
ηF_o = prestressing force after loss due to creep, etc.
h = over-all depth
M_G, M_L = bending moment due to dead load when the prestress is established, due to the additional superimposed dead and live load (bending moments due to vertical loads acting on a simply supported beam are taken as positive)
t = thickness of flanges of beam
y_b, y_t = distance of bottom, top fiber from centroid of concrete section
Z_b, Z_t = section modulus of bottom, top fiber

BALANCED DESIGN WITH CONSTANT WORKING STRESSES

If the maximum permissible stresses during the prestressing operation and under the action of superimposed loads are the same, a simple solution is obtained.

Denoting the maximum permissible concrete stresses in compression and tension by f_c and f_t, the inequalities (1) to (4)

$$\frac{F_o}{A_c} - \frac{F_o e}{Z_t} + \frac{M_G}{Z_t} = -f_t \quad (5)$$

$$\frac{F_o}{A_c} + \frac{F_o e}{Z_b} - \frac{M_G}{Z_b} = f_c \quad (6)$$

$$\frac{\eta F_o}{A_c} - \frac{\eta F_o e}{Z_t} + \frac{M_G}{Z_t} + \frac{M_L}{Z_t} = f_c \quad (7)$$

$$\frac{\eta F_o}{A_c} + \frac{\eta F_o e}{Z_b} - \frac{M_G}{Z_b} - \frac{M_L}{Z_b} = -f_t \quad (8)$$

Subtracting Eq. (5), multiplied by η, from Eq. (7)

$$(1 - \eta)\, M_G/Z_t + M_L/Z_t = f_c + \eta f_t$$

which gives the section modulus of the top fiber.

$$Z_t = \frac{(1 - \eta)\, M_G + M_L}{f_c + \eta f_t} \qquad (9)$$

Subtracting Eq. (8) from Eq. (6), multiplied by η

$$(1 - \eta)\, M_G/Z_b + M_L/Z_b = \eta f_c + f_t$$

which gives the section modulus of the bottom fiber.

$$Z_b = \frac{(1 - \eta)\, M_G + M_L}{\eta f_c + f_t} \qquad (10)$$

If y_t and y_b are the distance of the top and bottom fibers, respectively, from the centroid of the section

$$\frac{y_t}{y_b} = \frac{Z_b}{Z_t} = \frac{f_c + \eta f_t}{\eta f_c + f_t} \qquad (11)$$

Since f_c is normally more than ten times f_t, y_t is greater than y_b for balanced design, and the bottom flange is therefore larger than the top flange.

Subtracting Eq. (6) from Eq. (5)

$$F_o e\, (1/Z_t + 1/Z_b) - M_G\, (1/Z_t + 1/Z_b) = f_c + f_t$$

which gives

$$F_o e = M_G + (f_c + f_t)/(1/Z_t + 1/Z_b) \qquad (12)$$

Subtracting Eq. (7) from Eq. (8)

$$-\eta F_o e\, (1/Z_t + 1/Z_b) + (M_G + M_L)\, (1/Z_t + 1/Z_b) = f_c + f_t$$

which gives

$$\eta F_o e = M_G + M_L - (f_c + f_t)/(1/Z_t + 1/Z_b) \qquad (13)$$

Adding Eq. (12) and (13)

$$(1 + \eta)\, F_o e = 2\, M_G + M_L$$

which gives the product

$$F_o e = (2\, M_G + M_L)/(1 + \eta) \qquad (14)$$

Adding Eq. (5), multiplied by Z_t, to Eq. (6), multiplied by Z_b

$$(Z_b + Z_t)\, (F_o/A_c) = f_c Z_b - f_t Z_t \qquad (15)$$

Adding Eq. (7), multiplied by Z_t, to Eq. (8), multiplied by Z_b

$$(Z_b + Z_t)\, (\eta F_o/A_c) = f_c Z_t - f_t Z_t \qquad (16)$$

Adding Eq. (15) and (16)

$$(Z_b + Z_t)\, (1 + \eta)\, (F_o/A_c) = (f_c - f_t)\, (Z_b + Z_t)$$

which gives the ratio

$$F_o/A_c = (f_c - f_t)/(1 + \eta) \qquad (17)$$

Dividing Eq. (14) by Eq. (17), the product

$$e\, A_c = (2\, M_G + M_L)/(f_c - f_t) \qquad (18)$$

The section moduli are determined directly by Eq. (9) and (10), Z_b being greater than Z_t for balanced design with constant working stresses. Balanced design is therefore not possible for slabs or symmetrical sections, although the difference between a balanced and a symmetrical section is often slight.

The section moduli for balanced design having been calculated, Eq. (14) and (18) remain for the computation of the prestressing force F_o (which determines the steel area), the concrete area A_c (which determines the depth of the section), and the eccentricity e. As in normal design of reinforced concrete beams with compression steel, there are therefore a number of sections which satisfy the condition of balanced design, ranging from heavily reinforced shallow sections to lightly reinforced deep sections. In practice the choice is limited by the following considerations:

(1) The ratio of depth to width, h/b, is limited by the possible danger of torsion buckling.

(2) The ratio of rib thickness to over-all width, b'/b_b, is limited by the shear stresses, and the ratio of flange thickness to over-all depth, t/h, by secondary stresses in the flanges. The thickness of the rib and the flanges must be sufficient to allow correct placing of the concrete.

(3) The width of the top flange must be sufficient to ensure that the section has an adequate load factor against ultimate failure. A section with a narrow top flange and a large area of prestressing steel is liable to premature failure due to crushing of the concrete near the top face.

(4) Since the cables must normally lie inside the section, the eccentricity is limited by the depth of the section.

From Eq. (11), the distance of the bottom fiber of the section from the centroid

$$y_b = \frac{h}{y_t/y_b + 1} = \frac{\eta f_c + f_t}{(1 + \eta)(f_c + f_t)} h \quad \ldots\ldots (19)$$

If d_b denotes the distance of the centroid of the steel from the bottom fibers, the eccentricity of the prestressing force with regard to the centroid of the concrete section

$$e = y_b - d_b \quad \ldots\ldots (20)$$

Eq. (14) shows that the greatest steel economy is obtained if e is taken at its maximum value. The section should therefore be made as deep as possible and the steel placed close to its bottom face. As Eq. (18) shows, this satisfies also the condition of maximum economy in concrete and dead weight, since for a given section modulus the deepest section has the lowest cross-sectional area.

Example 1

A pre-tensioned concrete beam, simply supported at its ends, carries a uniformly distributed live load in addition to its own weight, the dead load and live load bending movements being 31,000 and 122,000 in.-lb, respectively. Calculate the midspan dimensions required for balanced design, if the maximum permissible steel stress is 160,000 psi, and the maximum permissible concrete stresses in compression and tension are 2000 and 130 psi, respectively. The loss of prestress on transfer may be taken as 15 percent, and a further 10 percent should be allowed for loss after transfer.

From Eq. (9) and (10) the section moduli

$$Z_t = \frac{0.10 \times 31{,}000 + 122{,}000}{2000 + 0.9 \times 130} = \frac{125{,}100}{2117} = 59.1 \text{ cu in.}$$

and $Z_b = \dfrac{125{,}100}{1930} = 64.8$ cu in.

From Eq. (14) and (18) the products

$$eF_c = \frac{2 \times 31{,}000 + 122{,}000}{1 + 0.90} = \frac{183{,}000}{1.90} = 96{,}300 \text{ in.-lb} \quad \text{(a)}$$

$$\text{and } eA_c = \frac{183{,}000}{2000 - 130} = 97.9 \text{ cu in.} \quad \text{(b)}$$

The ratio between the section moduli requires a value [from Eq. (19)]

$$y_b = \frac{1930}{1.9 \times 2130} h = 0.477\, h \quad \text{(c)}$$

Taking the distance of the steel from the bottom of the flange, $d_b = 0.10\, h$, the eccentricity $e \leq 0.377\, h$.

To ensure that the section is sufficiently deep to provide this eccentricity, the ratio

$$\frac{A_c/b_b h}{Z_t/b_b h^2} \geq \frac{97.9}{0.377 \times 59.1} = 4.39 \quad \text{(d)}$$

For balanced design the section must have the moduli calculated above, and in addition satisfy Eq. (a) to (d). Since the section, like most pre-tensioned sections, is small in size, it is essential to choose high ratios of b'/b_b and t/h to ensure an adequate thickness of the rib and flanges. The ratios $b_t/b_b = 0.8$; $b'/b_b = 0.5$; and $t/h = 0.2$ (section 74, Table 2) give:

$$y_b/h = 0.48 \text{ and } \frac{A_c/b_b h}{Z_t/b_b h^2} = \frac{0.66}{0.129} = 5.1$$

Using these ratios, the product $b_b h^2 = Z_t/0.129 = 458$. Taking the over-all width of the bottom flange $b_b = 4\frac{1}{2}$ in., the over-all depth $h = 458/4.5 = 10.17$ in. (say 10 in.). The width of the top flange $b_t = 0.8 \times 4.5 = 3.6$ in. (say $3\frac{3}{4}$ in.); the width of the rib $b' = 0.5 \times 4.5 = 2\frac{1}{4}$ in.; and the thickness of the flanges $t = 0.2 \times 10.17 = 2.03$ in. (say 2 in.).

The cross-sectional area of the concrete section $A_c = 2\,(3\frac{3}{4} + 4\frac{1}{2}) + 2\frac{1}{4}\,(10 - 2 \times 2) = 30.0$ sq in.; and the minimum eccentricity required for balanced design, from Eq. (b), $e = 97.9/30.0 = 3.26$ in.

The distance of the bottom flange from the centroid of the concrete section, $y_b = 0.48 \times 10 = 4.8$ in. The available distance from the center of the wires to the bottom of the flange is therefore $d_b = 4.8 - 3.3 = 1.5$ in., which is adequate.

From Eq. (a) the prestressing force required on transfer is $F_c = 96{,}300/3.26 = 29{,}500$ lb. The initial prestressing force $F_c/0.85 = 34{,}800$, and the steel area required $A_s = (F_c/0.85)/f_s = 34{,}800/160{,}000 = 0.217$ sq in.

BALANCED DESIGN WITH VARIABLE WORKING STRESSES

If the maximum permissible stresses for the prestressing operation and the working loads differ, a more elaborate solution is obtained. Denoting the maximum concrete stresses in compression and tension permissible during the prestressing operation by f_{cP} and f_{tP}, and the corresponding stresses under load by f_{cL} and f_{tL}, Eq. (5) to (8) become

$$\frac{F_c}{A_c} - \frac{F_c e}{Z_t} + \frac{M_G}{Z_t} = -f_{tP} \quad (21)$$

$$\frac{F_c}{A_c} + \frac{F_c e}{Z_b} - \frac{M_G}{Z_b} = f_{cP} \quad (22)$$

$$\frac{\eta F_c}{A_c} - \frac{\eta F_c e}{Z_t} + \frac{M_G}{Z_t} + \frac{M_L}{Z_t} = f_{cL} \quad (23)$$

$$\frac{\eta F_e}{A_c} + \frac{\eta F_e e}{Z_b} - \frac{M_G}{Z_b} - \frac{M_L}{Z_b} = -f_{tL} \quad (24)$$

Subtracting Eq. (21), multiplied by η, from Eq. (23)

$$(1 - \eta) M_G/Z_t + M_L/Z_t = f_{cL} + \eta f_{tP}$$

which gives the section modulus of the top fiber

$$Z_t = \frac{(1 - \eta) M_G + M_L}{f_{cL} + \eta f_{tP}} \quad (25)$$

Subtracting Eq. (24) from Eq. (22), multiplied by η, the section modulus of the bottom fiber

$$Z_b = \frac{(1 - \eta) M_G + M_L}{\eta f_{cP} + f_{tL}} \quad (26)$$

These relations are similar to those obtained in Eq. (9) and (10). Subtracting Eq. (22) from Eq. (21)

$$F_e e (1/Z_t + 1/Z_b) - M_G (1/Z_t + 1/Z_b) = f_{cP} + f_{tP}$$

which gives

$$F_e e = M_G + (f_{cP} + f_{tP})/(1/Z_t + 1/Z_b) \quad (27)$$

Subtracting Eq. (23) from Eq. (24)

$$-\eta F_e e (1/Z_t + 1/Z_b) + (M_G + M_L) (1/Z_t + 1/Z_b) = f_{cL} + f_{tL}$$

which gives

$$\eta F_e e = M_G + M_L - (f_{cL} + f_{tL})/(1/Z_t + 1/Z_b) \quad (28)$$

Adding Eq. (27) and (28)

$$(1 + \eta) F_e e = 2 M_G + M_L + (f_{cP} + f_{tP} - f_{cL} - f_{tL})/(1/Z_t + 1/Z_b) \quad (29)$$

From Eq. (25) and (26)

$$\frac{1}{Z_t} + \frac{1}{Z_b} = \frac{\eta (f_{cP} + f_{tP}) + (f_{cL} + f_{tL})}{(1 - \eta) M_G + M_L} \quad (30)$$

Substituting this expression into Eq. (29)

$$F_e e = \frac{(M_G + M_L) (f_{cP} + f_{tP}) + M_G (f_{cL} + f_{tL})}{\eta (f_{cP} + f_{tP}) + (f_{cL} + f_{tL})} \quad (31)$$

Adding Eq. (21), multiplied by Z_t, to Eq. (22), multiplied by Z_b

$$(Z_b + Z_t) (F_e/A_c) = f_{cP} Z_b - f_{tP} Z_t \quad (32)$$

Adding Eq. (23), multiplied by Z_t, to Eq. (24), multiplied by Z_b

$$(Z_b + Z_t) (\eta F_e/A_c) = f_{cL} Z_t - f_{tL} Z_b \quad (33)$$

Adding Eq. (32) and (33), and dividing by $Z_t Z_b$

$$(1 + \eta) \frac{F_e}{A_c} \left(\frac{1}{Z_t} + \frac{1}{Z_b} \right) = \frac{f_{cP} - f_{tL}}{Z_b} + \frac{f_{cL} - f_{tP}}{Z_b}$$

Substituting for Z_t, Z_b, and $1/Z_t + 1/Z_b$ from Eq. (25), (26), and (30)

$$(1 + \eta) \frac{F_e}{A_c} [\eta (f_{cP} + f_{tP}) + (f_{cL} + f_{tL})] = (f_{cP} - f_{tL}) (f_{cL} + \eta f_{tP}) + (f_{cL} - f_{tP}) (\eta f_{cP} + f_{tL})$$

which gives

$$\frac{F_e}{A_c} = \frac{f_{cP} f_{cL} - f_{tP} f_{tL}}{\eta (f_{cP} + f_{tP}) + (f_{cL} + f_{tL})} \quad (34)$$

Dividing Eq. (31) by Eq. (34)

$$eA_c = \frac{(M_G + M_L)(f_{cP} + f_{tP}) + M_G(f_{cL} + f_{tL})}{f_{cP}f_{cL} - f_{tP}f_{tL}} \quad \ldots\ldots (35)$$

To ensure that the cables or bars lie inside the section

$$e \leq y_b - d_b \quad \ldots\ldots (20)$$

From Eq. (19)

$$y_b = \frac{h}{y_t/y_b + 1} = \frac{h}{Z_b/Z_t + 1}$$

Substituting from Eq. (25) and (26)

$$y_b = \frac{\eta f_{cP} + f_{tL}}{\eta(f_{cP} + f_{tP}) + (f_{cL} + f_{tL})} h \quad \ldots\ldots (36)$$

If the live load can be divided into a stationary superimposed load and an occasional moving load, separate working stresses may be specified for the prestressing operation, for the stationary superimposed load only, and for the full load (including the moving load); under the action of the moving load a much higher concrete stress in tension may be admissible, and a small increase may be made in the maximum compressive stress.

Two separate designs are then required, one considering the loading conditions during the prestressing operation and under the action of the stationary superimposed load, and the other during the prestressing operation and under the action of the full load. It is frequently possible to tell from the relative magnitude of the bending moments involved which of the two designs is likely to be critical.

Example 2

A post-tensioned concrete beam, simply supported at its ends, carries a uniformly distributed stationary superimposed load and a moving load in addition to its own weight. The mid-span bending moments due to the dead load, the stationary superimposed load, and the moving load are 1,500,000, 1,875,000, and 1,500,000 in.-lb, respectively. Calculate the mid-span dimensions required for balanced design, if the maximum permissible steel stress is 135,000 psi, and the maximum permissible concrete stresses in compression and tension are 2400 and 130 psi, respectively, during the prestressing operation; 2000 and 0 psi, respectively, under the action of the stationary superimposed load only; and 2700 and 400 psi, respectively, under full load. The loss of prestress due to creep may be taken at 15 percent.

From Eq. (25) and (26) the section moduli for the stationary superimposed load only are

$$Z_t = \frac{0.15 \times 1{,}500{,}000 + 1{,}875{,}000}{2000 + 0.85 \times 130} = \frac{2{,}100{,}000}{2110} = 995 \text{ cu in.}$$

$$\text{and } Z_b = \frac{2{,}100{,}000}{0.85 \times 2400 + 0} = 1030 \text{ cu in.}$$

The section moduli for the full load are

$$Z_t = \frac{0.15 \times 1{,}500{,}000 + 3{,}375{,}000}{2700 + 0.85 \times 130} = \frac{3{,}600{,}000}{2810} = 1282 \text{ cu in.}$$

$$\text{and } Z_b = \frac{3{,}600{,}000}{0.85 \times 2400 + 400} = \frac{3{,}600{,}000}{2440} = 1476 \text{ cu in.}$$

The section must therefore be designed for its full load.

From Eq. (31) and (35) the products

$$eF_c = \frac{4{,}875{,}000 \times 2530 + 1{,}500{,}000 \times 3100}{0.85 \times 2530 + 3100} = \frac{16{,}970{,}000{,}000}{5250} = 3{,}240{,}000 \text{ in.-lb} \quad \text{(a)}$$

$$\text{and } eA_c = \frac{16{,}970{,}000{,}000}{2400 \times 2700 - 130 \times 400} = 2640 \text{ cu in.} \quad \text{(b)}$$

The ratio between the section moduli requires a value [from Eq. (35)]

$$y_b = \frac{0.85 \times 2400 + 400}{0.85 \times 2530 + 3100} h = 0.465\, h \quad \text{(c)}$$

Taking the distance of the steel from the bottom of the flange $d_b = 0.075\, h$ the eccentricity $e \leq 0.465\, h - 0.075\, h = 0.390\, h$.

To ensure that the section is sufficiently deep to provide this eccentricity

$$\frac{A_c/b_b\, h}{Z_t/b_b\, h^2} \geq \frac{2640}{0.39 \times 1282} = 5.27 \quad \text{(d)}$$

Since the section modulus is comparatively low, high values of b'/b and t/h are required to ensure adequate thickness of the rib and flanges. The ratios $b_t/b_b = 0.7$; $b'/b_b = 0.5$; and $t/h = 0.20$ (section 99, Table 2) give

$$y_b/h = 0.46 \text{ and } \frac{A_c/b_b\, h}{Z_t/b_b\, h^2} = 5.37$$

Using these ratios the product $b_b\, h^2 = Z_t/0.119 = 1282/0.119 = 10{,}780$ cu in. Taking the flange thickness $t = 6$ in., and the over-all depth $h = 30$ in.; the net width of the bottom flange $b_b = 10{,}780/30^2 = 11.98$ in. (say 12 in.); the width of the top flange $b_t = 0.7$; $b = 8.39$ in., say 9 in.; and the thickness of the rib $b' = 0.5$; $b = 5.99$ in., say 6 in.

The cross-sectional area of the concrete section $A_c = 6 \times 30 + (9 - 6 + 12 - 6) \times 6 = 234$ sq in., and the minimum eccentricity required for balanced design, from Eq. (b) $e = 2640/234 = 11.28$ in.

The distance of the bottom fiber from the centroid of the concrete section, $y_b = 0.46 \times 30 = 13.8$ in. The available distance from the center of the cables to the bottom of the flange is therefore $d_b = 13.8 - 11.3 = 2.5$ in., which is adequate.

From Eq. (a) the steel area required $A_s = 3{,}240{,}000/11.28 \times 135{,}000 = 2130$ sq in.

CONCLUSION

Since there are only four simultaneous equations for computing the five quantities required for the design of an unsymmetrical prestressed concrete section, there is an infinite number of solutions, ranging from heavily reinforced shallow sections to lightly reinforced deep sections. It is noteworthy that for balanced design an increase in depth reduces both the amount of steel *and* the amount of concrete required. In practice this will, however, frequently yield beams which are excessively thin and narrow, and a thickening or widening of the concrete section may then be required to achieve the desired steel economy. The balanced design is therefore particularly useful for the more heavily loaded beams, in which the concrete section is necessarily substantial. The balanced design also represents the ideal solution when the over-all depth is restricted, or when it is desired to reduce the dead weight due to the concrete composing the beam.

In the light building construction the smallest area of steel is generally obtained from designs which are not balanced. In this respect prestressed concrete I-sections resemble normal reinforced concrete T-beams, in which

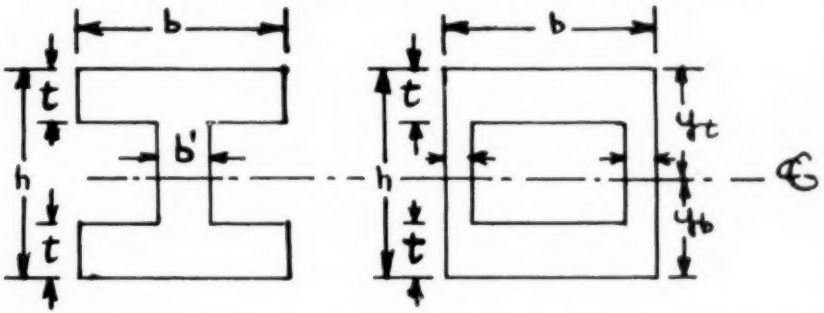
the greatest steel economy is generally achieved by the use of under-reinforced designs.* The balanced design, however, represents a useful starting point even when it is not the most economical solution, since the section moduli obtained from it are the lowest satisfying the basic inequalities (1) to (4), with which every design must conform.

ACKNOWLEDGMENT

Since this paper was composed, papers by K. P. Millbradt and T. J. Kofodimos ("Design of Monolithic Concrete Frame—Prestressed," *Proceedings*, ASCE, Separate No. 217, June 1953) and by T. Y. Lin and A. C. Scordelis ("Selection and Design of Prestressed Concrete Beam Sections," ACI JOURNAL, Nov. 1953, *Proc.* V. 50, pp. 209-224) have been published, which propose design methods along similar lines to those outlined by the author.

*Cowan, Henry J., author's closure to discussion of "Direct Design of T-Beams," ACI JOURNAL, Part 2, Dec. 1951, *Proc.* V. 47, p. 544-3.

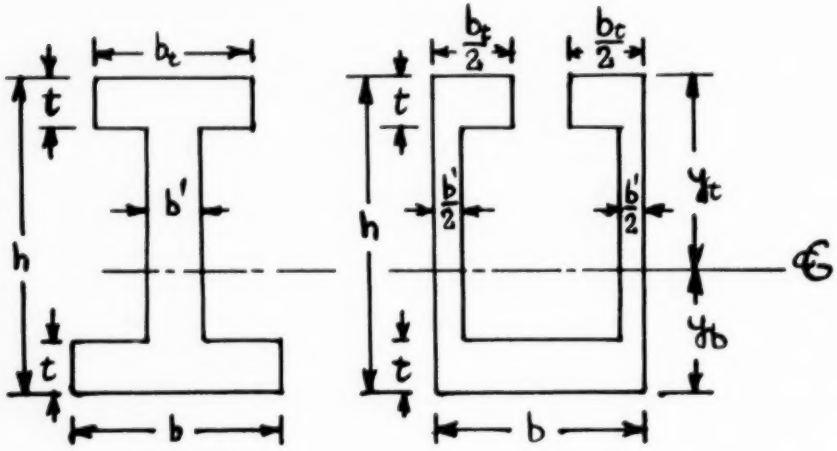
TABLE 1—PROPERTIES OF SYMMETRICAL I- AND BOX-SECTIONS



No.	b'/b	t/h	A_c/bh	I_c/bh^3	Z_t/bh^2 and Z_b/bh^2
1	0.1	0.05	0.19 9	0.029 16	0.057 33
2		0.10	0.28 9	0.045 13	0.080 25
3		0.15	0.37 9	0.058 9	0.115 19
4		0.20	0.46 9	0.067 7	0.134 14
5		0.25	0.55	0.074	0.148
6	0.2	0.05	0.28 8	0.035 14	0.069 29
7		0.10	0.36 8	0.049 11	0.098 23
8		0.15	0.44 8	0.060 9	0.121 17
9		0.20	0.52 8	0.069 6	0.138 12
10		0.25	0.60	0.075	0.150
11	0.3	0.05	0.37 7	0.041 12	0.081 26
12		0.10	0.44 7	0.053 10	0.107 20
13		0.15	0.51 7	0.063 8	0.127 15
14		0.20	0.58 7	0.071 5	0.142 10
15		0.25	0.65	0.076	0.152
16	0.4	0.05	0.46 6	0.047 11	0.094 22
17		0.10	0.52 6	0.058 9	0.116 16
18		0.15	0.58 6	0.067 6	0.132 13
19		0.20	0.64 6	0.073 4	0.145 9
20		0.25	0.70	0.077	0.154
21	0.5	0.05	0.55 5	0.053 9	0.106 18
22		0.10	0.60 5	0.062 7	0.124 14
23		0.15	0.65 5	0.069 5	0.138 10
24		0.20	0.70 5	0.074 4	0.148 8
25		0.25	0.75 5	0.078	0.156

TABLE 2—PROPERTIES OF UNSYMMETRICAL I-SECTIONS, BULB T-SECTIONS, AND BULB U-SECTIONS

(Long flange at bottom face)



No.	b_t/b	b'/b	t/h	A_c/bh	y_c/h	y_t/h	I_c/bh^3	Z_b/bh^2	Z_t/bh^2
26	0.9	0.1	0.05	0.19 8	0.49 1	0.51 1	0.027 16	0.056 32	0.053 30
27			0.10	0.27 9	0.48 0	0.52 0	0.043 12	0.088 25	0.083 22
28			0.15	0.36 8	0.48 0	0.52 0	0.055 8	0.113 18	0.105 17
29			0.20	0.44 9	0.48 0	0.52 0	0.063 7	0.131 14	0.122 13
30			0.25	0.53	0.48	0.52	0.070	0.145	0.135
31		0.2	0.05	0.28 7	0.49 0	0.51 0	0.034 13	0.069 28	0.066 26
32			0.10	0.35 8	0.49 0	0.51 0	0.047 11	0.097 22	0.092 20
33			0.15	0.43 7	0.49 1	0.51 0	0.058 8	0.119 17	0.112 15
34			0.20	0.50 7	0.48 1	0.52 1	0.066 8	0.136 17	0.127 15
35			0.25	0.58 8	0.48 0	0.52 0	0.072 6	0.148 12	0.138 11
36		0.3	0.05	0.37 6	0.49 0	0.51 0	0.040 11	0.080 24	0.078 23
37			0.10	0.43 7	0.49 0	0.51 0	0.051 9	0.104 19	0.101 17
38			0.15	0.50 6	0.49 0	0.51 0	0.060 7	0.123 14	0.118 13
39			0.20	0.56 7	0.49 0	0.51 0	0.067 5	0.137 10	0.131 9
40			0.25	0.63	0.49	0.51	0.072	0.147	0.140
41		0.4	0.05	0.46 5	0.49 0	0.51 0	0.046 10	0.094 20	0.090 20
42			0.10	0.51 6	0.49 0	0.51 0	0.056 8	0.114 16	0.110 14
43			0.15	0.57 5	0.49 0	0.51 0	0.064 5	0.130 12	0.124 11
44			0.20	0.62 6	0.49 0	0.51 0	0.069 4	0.142 8	0.135 8
45			0.25	0.68	0.49	0.51	0.073	0.150	0.143
46		0.5	0.05	0.55 4	0.49 0	0.51 0	0.052 8	0.105 17	0.102 16
47			0.10	0.59 5	0.49 0	0.51 0	0.060 6	0.122 13	0.118 12
48			0.15	0.64 4	0.49 0	0.51 0	0.066 5	0.135 10	0.130 9
49			0.20	0.68 5	0.49 0	0.51 0	0.071 4	0.145 8	0.139 7
50			0.25	0.73	0.49	0.51	0.075	0.153	0.146
51	0.8	0.1	0.05	0.18 8	0.47 0	0.53 0	0.026 15	0.056 31	0.050 26
52			0.10	0.26 8	0.47 1	0.53 1	0.041 11	0.087 24	0.076 20
53			0.15	0.34 8	0.46 0	0.54 0	0.052 8	0.112 19	0.096 16
54			0.20	0.42 8	0.46 0	0.54 0	0.060 6	0.130 13	0.112 11
55			0.25	0.52	0.46	0.54	0.066	0.143	0.123
56		0.2	0.05	0.27 7	0.48 1	0.52 1	0.032 13	0.067 28	0.062 23
57			0.10	0.34 7	0.47 0	0.53 0	0.045 10	0.095 22	0.085 18
58			0.15	0.41 7	0.47 0	0.53 0	0.055 7	0.117 16	0.103 13
59			0.20	0.48 7	0.47 0	0.53 0	0.062 5	0.133 11	0.116 10
60			0.25	0.55	0.47	0.53	0.067	0.144	0.126
61		0.3	0.05	0.36 6	0.49 1	0.51 1	0.038 11	0.079 24	0.075 19
62			0.10	0.42 6	0.48 1	0.52 1	0.049 9	0.103 19	0.094 15
63			0.15	0.48 6	0.47 0	0.53 0	0.058 6	0.122 14	0.109 11
64			0.20	0.54 6	0.47 0	0.53 0	0.064 4	0.136 10	0.120 8
65			0.25	0.60	0.47	0.53	0.068	0.146	0.128

TABLE 2 (Cont'd)

No.	b_0/b	b'/b	t/h	A_c/bh	y_b/h	y_t/h	I_c/bh^3	Z_b/bh^2	Z_t/bh^2
66		0.4	0.05	0.45 5	0.49 1	0.51 1	0.045 8	0.091 20	0.087 16
67			0.10	0.50 5	0.48 0	0.52 0	0.053 7	0.111 16	0.103 12
68			0.15	0.55 5	0.48 1	0.52 1	0.060 7	0.127 16	0.115 12
69			0.20	0.60 5	0.47 1	0.53 1	0.065 5	0.139 12	0.124 9
70			0.25	0.65 5	0.47 0	0.53 0	0.069 4	0.147 8	0.131 7
71		0.5	0.05	0.54 4	0.49 1	0.51 1	0.051 7	0.104 16	0.099 13
72			0.10	0.58 4	0.48 0	0.52 0	0.058 5	0.120 12	0.112 10
73			0.15	0.62 4	0.48 0	0.52 0	0.063 4	0.132 9	0.122 7
74			0.20	0.66 4	0.48 1	0.52 1	0.067 3	0.141 8	0.129 5
75			0.25	0.70 4	0.47 1	0.53 1	0.070	0.149	0.134
76	0.7	0.1	0.05	0.18 7	0.46 1	0.54 1	0.025 13	0.055 30	0.046 23
77			0.10	0.25 8	0.45 1	0.55 1	0.038 10	0.085 24	0.069 18
78			0.15	0.33 7	0.44 0	0.56 0	0.048 8	0.109 18	0.087 13
79			0.20	0.40 8	0.44 0	0.56 0	0.056 6	0.127 14	0.100 10
80			0.25	0.48	0.44	0.56	0.061 5	0.138 11	0.108 8
81		0.2	0.05	0.27 6	0.47 1	0.53 1	0.031 12	0.066 27	0.059 20
82			0.10	0.33 7	0.46 1	0.54 1	0.043 9	0.093 21	0.079 15
83			0.15	0.40 6	0.45 0	0.55 0	0.052 6	0.114 15	0.094 11
84			0.20	0.46 7	0.45 0	0.55 0	0.058 5	0.129 11	0.105 8
85			0.25	0.53	0.45	0.55	0.063	0.140	0.113
86		0.3	0.05	0.36 5	0.48 1	0.52 1	0.037 10	0.078 22	0.072 16
87			0.10	0.41 6	0.47 1	0.53 1	0.047 7	0.100 18	0.088 12
88			0.15	0.47 5	0.46 1	0.54 1	0.054 6	0.118 14	0.100 9
89			0.20	0.52 5	0.45 1	0.55 0	0.060 4	0.132 10	0.109 7
90			0.25	0.58 6	0.45	0.55	0.064	0.142	0.116
91		0.4	0.05	0.45 4	0.48 1	0.52 1	0.043 8	0.089 19	0.084 13
92			0.10	0.49 4	0.47 1	0.53 1	0.051 6	0.108 15	0.097 10
93			0.15	0.54 5	0.46 1	0.54 0	0.057 5	0.123 12	0.107 7
94			0.20	0.58 4	0.46 0	0.54 1	0.062 3	0.135 9	0.114 5
95			0.25	0.63 5	0.45	0.55	0.065	0.143 8	0.119
96		0.5	0.05	0.54 3	0.49 1	0.51 1	0.049 7	0.102 15	0.096 10
97			0.10	0.57 4	0.48 1	0.52 1	0.056 5	0.117 12	0.106 8
98			0.15	0.61 3	0.47 1	0.53 1	0.061 3	0.129 9	0.114 5
99			0.20	0.64 4	0.46 1	0.54 0	0.064 2	0.138 7	0.119 3
100			0.25	0.68	0.46	0.54	0.066	0.145	0.122
101	0.6	0.1	0.05	0.17 7	0.44 1	0.56 1	0.024 11	0.055 28	0.043 18
102			0.10	0.24 7	0.43 1	0.57 1	0.035 9	0.083 23	0.064 15
103			0.15	0.31 7	0.42 1	0.58 0	0.044 7	0.106 17	0.076 12
104			0.20	0.38 7	0.42 0	0.58 0	0.051 6	0.123 17	0.088 9
105			0.25	0.45	0.42	0.58	0.057	0.134	0.097
106		0.2	0.05	0.26 6	0.46 2	0.54 2	0.030 10	0.065 26	0.056 16
107			0.10	0.32 6	0.44 1	0.56 1	0.040 8	0.091 20	0.072 12
108			0.15	0.38 6	0.43 1	0.57 0	0.048 5	0.111 14	0.084 9
109			0.20	0.44 6	0.43 0	0.57 0	0.053 4	0.125 9	0.093 7
110			0.25	0.50 6	0.43	0.57	0.057	0.134	0.100
111		0.3	0.05	0.35 5	0.47 2	0.53 2	0.036 9	0.076 22	0.068 14
112			0.10	0.40 5	0.45 1	0.55 1	0.045 6	0.098 17	0.082 10
113			0.15	0.45 5	0.44 1	0.56 0	0.051 5	0.115 14	0.092 7
114			0.20	0.50 5	0.44 0	0.56 1	0.056 3	0.127 12	0.099 5
115			0.25	0.55 5	0.43 1	0.57 1	0.059	0.136 9	0.104
116		0.4	0.05	0.44 4	0.48 2	0.52 2	0.042 7	0.088 18	0.080 11
117			0.10	0.48 4	0.46 1	0.54 1	0.049 5	0.106 14	0.091 7
118			0.15	0.52 4	0.45 1	0.55 1	0.054 4	0.120 10	0.098 4
119			0.20	0.56 4	0.44 1	0.56 0	0.058 2	0.130 7	0.104 3
120			0.25	0.60 4	0.44 0	0.56 0	0.060 2	0.137 7	0.107 3
121		0.5	0.05	0.53 3	0.48 1	0.52 1	0.048 5	0.100 14	0.093 7
122			0.10	0.56 3	0.47 1	0.53 1	0.053 4	0.114 11	0.100 5
123			0.15	0.59 3	0.46 1	0.54 1	0.057 3	0.125 8	0.105 3
124			0.20	0.62 3	0.45 1	0.55 1	0.060 2	0.133 8	0.108 2
125			0.25	0.65 3	0.44 1	0.56 1	0.062 2	0.139 6	0.110 2
126	0.5	0.1	0.05	0.17 6	0.43 2	0.57 2	0.022 10	0.051 27	0.039 15
127			0.10	0.23 7	0.41 2	0.59 2	0.032 8	0.078 22	0.054 12
128			0.15	0.30 6	0.39 2	0.61 2	0.040 6	0.100 17	0.066 10
129			0.20	0.36 6	0.39 0	0.61 0	0.046 4	0.117 14	0.076 7
130			0.25	0.43 7	0.39 0	0.61 0	0.050	0.128 11	0.083
131		0.2	0.05	0.26 5	0.45 2	0.55 2	0.029 8	0.063 24	0.052 13
132			0.10	0.31 6	0.43 2	0.57 2	0.037 7	0.087 20	0.065 10
133			0.15	0.37 5	0.41 2	0.59 1	0.044 5	0.107 14	0.075 7
134			0.20	0.42 6	0.40 1	0.60 0	0.049 3	0.121 9	0.082 5
135			0.25	0.48 6	0.40 0	0.60 0	0.052	0.130	0.087

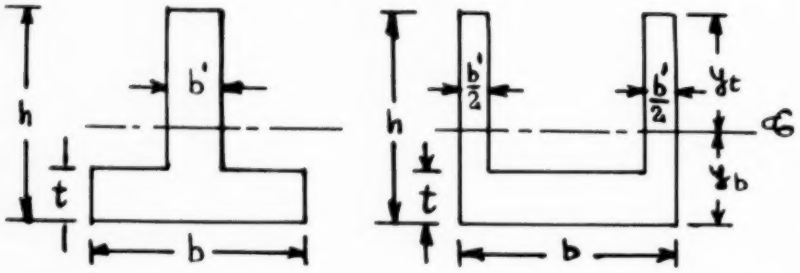
Continued on p. 784

TABLE 2 (Cont'd)

No.	b_t/b	b'/b	t/h	A_c/bh	y_b/h	y_t/h	I_c/bh^3	Z_b/bh^2	Z_t/bh^2
136		0.3	0.05	0.35 4	0.47 2	0.53 2	0.035 7	0.074 20	0.065 10
137			0.10	0.39 5	0.45 2	0.55 2	0.042 5	0.094 16	0.075 7
138			0.15	0.44 4	0.43 1	0.57 1	0.047 4	0.110 12	0.082 5
139			0.20	0.48 5	0.42 1	0.58 1	0.051 3	0.122 9	0.087 4
140			0.25	0.53	0.41	0.59	0.054	0.131	0.091
141		0.4	0.05	0.44 3	0.48 2	0.52 2	0.041 5	0.086 16	0.078 7
142			0.10	0.47 4	0.46 2	0.54 2	0.046 4	0.102 13	0.085 5
143			0.15	0.51 3	0.44 1	0.56 1	0.050 3	0.115 10	0.090 3
144			0.20	0.54 4	0.43 1	0.57 1	0.053 2	0.125 7	0.093 2
145			0.25	0.58	0.42	0.58	0.055	0.132	0.095
146		0.5	0.05	0.53 2	0.48 2	0.52 2	0.047 4	0.098 13	0.090 4
147			0.10	0.55 3	0.46 2	0.54 2	0.051 3	0.111 10	0.094 3
148			0.15	0.58 2	0.44 0	0.56 0	0.054 1	0.121 6	0.097 1
149			0.20	0.60 3	0.44 1	0.56 1	0.055 1	0.127 5	0.098 0
150			0.25	0.63	0.43	0.57	0.056	0.132	0.098

TABLE 3—PROPERTIES OF T-SECTIONS AND U-SECTIONS

(Flange on bottom face)



No.	b'/b	t/h	A_c/bh	y_b/h	y_t/h	I_c/bh^3	Z_b/bh^2	Z_t/bh^2
151	0.1	0.05	0.15 4	0.35 6	0.65 6	0.015 3	0.042 20	0.023 2
152		0.10	0.19 5	0.29 3	0.71 3	0.018 1	0.062 12	0.025 0
153		0.15	0.24 4	0.26 2	0.74 2	0.019 0	0.074 5	0.025 0
154		0.20	0.28 5	0.24 2	0.76 2	0.019 0	0.079 1	0.025 0
155		0.25	0.33	0.24 0	0.76 0	0.019	0.080	0.025
156	0.2	0.05	0.24 4	0.42 5	0.58 5	0.024 4	0.057 20	0.042 3
157		0.10	0.28 4	0.37 3	0.63 3	0.028 2	0.077 12	0.045 1
158		0.15	0.32 4	0.34 2	0.66 2	0.030 1	0.089 8	0.046 0
159		0.20	0.36 4	0.32 1	0.68 1	0.031 0	0.097 4	0.046 0
160		0.25	0.40	0.31	0.69	0.031	0.101	0.046
161	0.3	0.05	0.34 3	0.45 4	0.55 4	0.032 4	0.071 18	0.058 4
162		0.10	0.37 4	0.41 2	0.59 2	0.036 3	0.089 12	0.062 2
163		0.15	0.41 3	0.39 2	0.61 2	0.039 1	0.101 8	0.064 1
164		0.20	0.44 4	0.37 1	0.63 1	0.040 1	0.109 5	0.065 0
165		0.25	0.48	0.36 1	0.64 1	0.041 1	0.114	0.065
166	0.4	0.05	0.43 3	0.47 3	0.53 3	0.040 4	0.085 15	0.074 4
167		0.10	0.46 3	0.44 2	0.56 2	0.044 3	0.100 11	0.078 3
168		0.15	0.49 3	0.42 1	0.58 1	0.047 1	0.111 8	0.081 1
169		0.20	0.52 3	0.41 1	0.59 1	0.048 1	0.119 5	0.082 0
170		0.25	0.55	0.40 1	0.60 1	0.049	0.124	0.082
171	0.5	0.05	0.53 2	0.48 2	0.52 2	0.047 4	0.098 13	0.090 4
172		0.10	0.55 3	0.46 2	0.54 2	0.051 3	0.111 10	0.094 3
173		0.15	0.58 2	0.44 0	0.56 0	0.054 1	0.121 6	0.097 1
174		0.20	0.60 3	0.44 1	0.56 1	0.055 1	0.127 5	0.098 0
175		0.25	0.63	0.43	0.57	0.056	0.132	0.098

Title No. 51-40

An Interpretation of Some Published Researches on the Alkali-Aggregate Reaction*

Part 2—A Hypothesis Concerning Safe and Unsafe Reactions with Reactive Silica in Concrete

By T. C. POWERS and H. H. STEINOUR†

SYNOPSIS

Enlarging on considerations set down in Part 1,[1] the authors develop a hypothesis concerning reactions between reactive silica, calcium hydroxide, and alkali in concrete. The hypothesis is that the reaction proceeds safely or unsafely according to the relative amounts of calcium hydroxide and alkali adsorbed in the outer part of the reacted layer of the opal particle. Published experimental data are analyzed according to the hypothesis with the aid of Kalousek's data on the lime-alkali-silica-water system. Several conclusions are drawn, bearing on safe limits of alkali content and safe silica-alkali ratios.

INTRODUCTION

In Part 1 of this paper[1] we discussed indications found in published research concerning the nature of reactive aggregate, the chemistry of the reactions between calcium hydroxide, alkali, and reactive silica, and the mechanism of expansion. In the present paper we attempt an application of some of the ideas concerning the chemistry of the reactions to certain experimental data, paying particular attention to the factors that determine the safe amounts of alkali when reactive silica is present as opal.

ALKALI NULLIFIED BY REACTIVE AGGREGATE

Concrete can contain a certain amount of alkali without developing expansion even though reactive aggregate is present. The safe‡ quantity is not a fixed fraction of the amount of cement. It depends mainly on the amount of reactive mineral or on the total boundary area of reactive mineral or on both of these factors.

This conclusion is required by relationships brought out in various experimental studies reported in the literature. Consider, for example, the data shown in Fig. 1 due to Vivian.[2] (*Table 2*) This gives the expansion of mortar

*Received by the Institute July 2, 1954. Title No. 51-40 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Aug. 1, 1955. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†Members American Concrete Institute, Manager, Basic Research, and Manager, Literature Research Section, respectively, Portland Cement Assn., Chicago, Ill.

‡"Safe" is used here, and later, in the sense of not producing appreciable expansion. In some usages the sense may be relative to the prevailing circumstances, as may be inferred from the context.

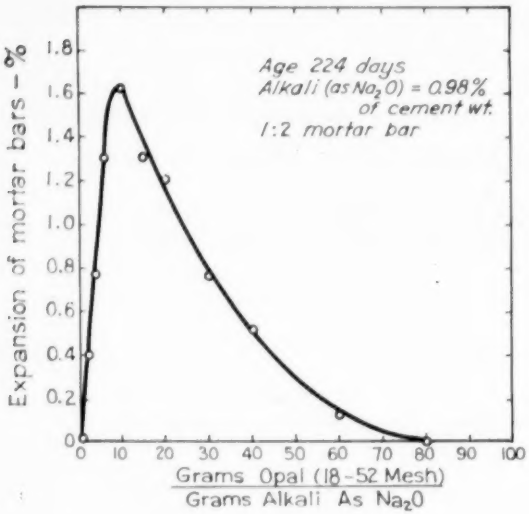


Fig. 1—Typical relationship between expansion and amount of opal particles in the test specimen (Data from Vivian)

bars in relation to the amount of opal in the mix when using a cement containing about 1 percent alkali. The relationship shown is typical of that found originally by Stanton[3] and later by others. The amount of expansion is a complicated function of the quantity of reactive aggregate, in this case opal. If the quantity of opal exceeds a certain amount, 80 g per g of alkali in Fig. 1, no expansion occurs. Thus, an amount of alkali equal to 1 percent of the weight of the cement is not safe for certain amounts of opal but is safe for relatively large amounts.

When the reactive mineral is powdered, it can be used in a wide range of proportions without causing expansion. This was demonstrated by Vivian.[4] When opal was ground to pass the No. 300 sieve and used in several proportions, expansion was practically zero for all proportions. The particle size of the reactive mineral is clearly an important factor.

A commercial aggregate containing reactive mineral will be able to tolerate a certain amount of alkali. An example is shown in Fig. 2. This shows the amounts of expansion obtained when using a certain California sand, called Saticoy, in relation to the alkali content of the cement.[5] Evidently, alkali up to 0.45 percent was safe with this aggregate in this mix.

When an aggregate contains reactive mineral and is used with an amount of alkali greater than it can tolerate, expansion can be prevented by adding an appropriate amount of pulverized reactive mineral, as has been shown by Hanna,[16] Stanton,[5] and others. An example is shown in Fig. 3. This shows how the expansion of Saticoy sand when used with high-alkali cement could be controlled by adding pulverized opaline chert. Expansion was nil after 12 months' storage at 130 F when the amount of pulverized chert was a little over 11 percent of the weight of the cement, or approximately 10 times the weight of the alkali oxide in the mix.

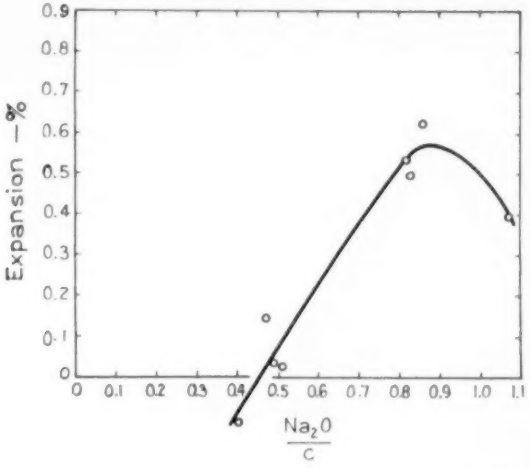


Fig. 2—Relation of expansion to alkali content of the cement for Saticoy sand (Data from Reference 5, Table V, Saticoy sand, age 12 months, 130 F)

D. O. Woolf[6] published data somewhat like those in Fig. 2. His data are especially valuable because they pertain to aggregates in which the kind, amount, and size of reactive particles are known. The mixes were 1:2 Ottawa sand mortars in which different amounts of Ottawa sand were replaced by particles of opal (16 to 30 mesh). Curves like those in Fig. 2 indicated rather definitely the average safe amounts of alkali for different percentages of opal in the sand. The data shown in Table 1 were taken from Woolf's Fig. 4 and are based on expansions at 18 months.

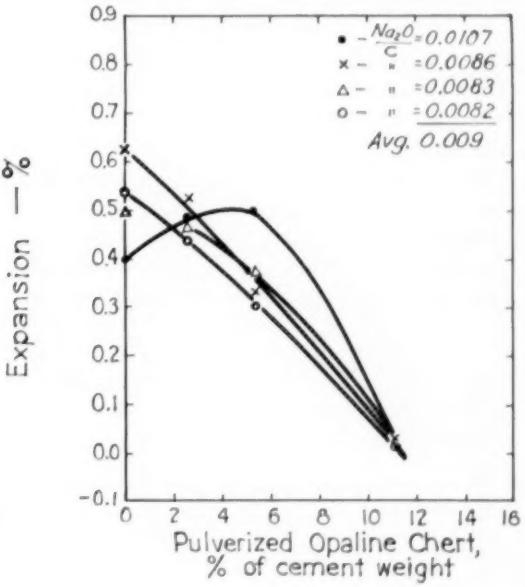


Fig. 3—Effect of pulverized opaline chert on expansion (Data from Reference 5, Table V, Saticoy sand, age 12 months, 130 F)

TABLE 1—SAFE ALKALI CONTENTS OF CEMENT FOR DIFFERENT PERCENTAGES OF OPAL IN SAND*

Item	Percent opal in sand	Observed safe alkali content of cement, percent	
		For no expansion	For 0.1 percent expansion
1	1	0.15 ± 0.05	0.19
2	2	0.25	0.30
3	5	0.50	0.65

*w/c = 0.5 by weight.

Data published earlier by Davis[7] gave similar results. Davis found that when opal particles constituted 5 percent of the aggregate (mix 1:2) no expansion occurred with cements containing alkali up to 0.48 percent. With a cement containing 0.61 percent alkali, expansion occurred. When opal constituted 2½ percent of the aggregate, expansion occurred with a cement containing 0.37 percent alkali.

THEORETICAL CONSIDERATIONS

We have just reviewed experimental evidence showing that opal particles and alkali can exist together in cement mortar in some combinations without producing expansion, whereas for other combinations expansion does occur, and indeed may occur at alkali percentages that are apparently quite safe under different circumstances. It might be thought that lack of expansion means lack of alkali reaction, but this does not appear to be the case; reaction between the alkali and silica has been shown under circumstances where expansion did not occur.[8] Our problem is to devise a theory, or hypothesis, that will account for these facts. We shall endeavor to show that it is theoretically possible for all the silica in an aggregate to be converted to nonexpansive lime-alkali-silica complex when conditions are such that lime can reach the reaction site (the unreacted core of a silica particle) and react with initially formed alkali-silica complex before swelling can occur. We shall present the view that under certain circumstances surface diffusion of adsorbed lime through the already reacted part of a silica particle can enable the lime to reach the site in adequate amount. However, we shall show also that the higher the alkali concentration, and the thicker the reaction layer, the more difficult it should be for the lime to reach the reaction site. Failure of lime transport should then permit the expansive reaction to develop. Thus, according to our hypothesis the alkali reacts safely under some circumstances, and under others it does not. In this section we shall develop this hypothesis in further detail through consideration of the requirements to be met.

Initial reactions

We can assume that the initial attack produces one of the lime-alkali-silica complexes indicated in Table 2, reproduced from Part 1, which presents

TABLE 2—SOME OF KALOUSEK'S DATA ON THE EQUILIBRIUM PRODUCTS OF MIXTURES OF SODA, LIME, SILICA, AND WATER WHEN THE LIQUID PHASE IS SATURATED WITH CALCIUM HYDROXIDE

Composition of solution, g per liter			Composition of solid phase in equilibrium with solution, molar ratios			
Na_2O	CaO	SiO_2	Na_2O	CaO*	SiO_2	H_2O
0.20	0.912	0.003	0.003	1.98	1.00	3.2
2.4	0.237	0.003	0.04	1.47	1.00	3.3
5.5	0.100	0.005	0.07	1.32	1.00	3.0
9.2	0.039	0.012	0.14	1.18	1.00	2.9
11.6	0.023	0.014	0.19	1.05	1.00	2.7
22.3	0.016	0.071	0.21	1.00	1.00	2.6
50.9	0.01	0.178	0.22	0.97	1.00	2.7
75.2	0.01	0.220	0.24	1.03	1.00	2.9
100.6	0.01	0.498	0.25	1.04	1.00	2.6

*These are Kalousek's computed values. The values for the other oxides were obtained by analysis.

experimental data obtained by Kalousek.[9] Even though the amount of lime in solution is very small relative to the amount of alkali, the solution is saturated with calcium hydroxide, for abundant solid calcium hydroxide is close at hand. Indeed, at the start the opal particles may be partly covered with calcium hydroxide crystals. The initial reaction is therefore supposed to produce a thin layer of lime-alkali-silica complex that is incapable of swelling.

Once formed, this layer separates the unreacted silica from the supply of lime, alkali, and water in the hardened portland cement paste surrounding the opal particle. The outer boundary of the layer is thus in contact with lime, alkali, and water, and the inner boundary, with unreacted silica. Hence, the composition of the outer part will tend to be such as to be in equilibrium with the contiguous solution, and the inner part will be poorer in lime and alkali than the outer part. The outer part of the layer tends to lose lime and alkali to the inner part, and the inner part of the layer tends to lose lime and alkali to the unreacted silica. As the outer part of the layer is thus depleted, it receives lime and alkali from the outside solution.

Progressive reactions

The reaction at the inner boundary of the layer is, according to Vivian,[10] a severing of the silicon-oxygen-silicon bonds by OH ions, accompanied by a reaction with sodium and calcium ions. The product is a complex having characteristics depending on the relative amounts of lime and alkali available, as already described; it may or may not be a swelling gel, depending on the amount of lime it contains.[1]

We do not know the minimum amount of lime required to produce a nonswelling gel. Probably the amount is less than the least amount found by Kalousek in the lime-alkali-silica complexes formed in solutions saturated with calcium hydroxide. We believe that the minimum lime requirement is less, principally because a nonswelling product does form under some conditions and because the solution at the reaction site cannot be saturated with calcium hydroxide during the reaction, except at the time of the initial attack and perhaps at the end of the reaction. Thus we proceed on the

assumption (based on experimental observation) that when conditions are right, lime can be delivered to the reaction site at the rate required for a safe reaction.

We might suppose that the relative amounts of lime and alkali reaching the reaction site in a given time depend on their relative concentrations in the solutions in the gel pores of the reaction layer. There *is* a connection, but, in our opinion, it is primarily an indirect one. For, if we think only in terms of relative rates of transfer through the solution phase, it is difficult to account for the fact that safe reactions do sometimes occur. The amounts of lime and alkali in solution in the gel pores of the reaction layer cannot be higher than the amounts in the solution outside the reaction layer. From the work of Kalousek and from the experiments described in the appendix to the preceding paper, we know that even when the concrete is made with a low-alkali cement and no expansion occurs, there may be 50 times as much alkali as lime in solution, and yet a safe product probably contains more lime than alkali. Thus, ordinary diffusion of dissolved ions does not seem to provide a basis for a satisfactory explanation. The key to understanding how a sufficient amount of lime reaches the reaction site seems to be a knowledge of the composition and physical properties of the lime-alkali-silica complexes.

Properties of lime-alkali-silica complex

We have evidence that the lime-alkali-silica complex formed in the presence of excess calcium hydroxide is a gel* similar to but not identical with the gel produced by hydration of portland cement. Therefore, we believe that the gel produced by the action of lime and alkali on opal is a porous material, the boundary area of the pores being very large compared with the pore volume. Such a material is permeable to water or aqueous solutions. As already said, the outer part of the layer may be assumed to be one of the complexes found by Kalousek, when the solution was saturated with lime. As shown in Table 2, Kalousek found that under this condition the solid phase always contained at least one mole of lime to one of silica. It contained only one mole of lime for all concentrations of Na_2O in solution above 20 g per liter. For lower concentrations of alkali, the amount of lime in the solid phase increased, not stepwise as in a chemical reaction, but continuously with decrease in alkali concentration. The amount of CaO approached a maximum of 2 moles as the alkali in solution approached zero. We think it a plausible assumption that when alkali is present the solid phase is a monocalcium silicate gel holding additional adsorbed lime and adsorbed alkali. In any case, Kalousek's empirical data can be handled as if the mechanism is adsorption, whether it is or not. Whatever the true state of combination in the gel, there is evidently a large internal surface.† Whether the atoms

*In this discussion the term "gel" usually refers to the white solid that is the lime-alkali-silica complex. It does not refer to the jelly-like exudations and deposits often described in connection with this subject, although these too may be gels.

†A gel is composed of submicroscopic particles. The internal surface referred to is the surface area of these particles. Or, the internal surface may be thought of as the wall area of the gel pores, the pores being interstitial spaces among the gel particles.

or ions exposed at a surface are held there by physical or chemical forces, they obviously have superior possibilities for readjustment to altered conditions.

Surface diffusion

Diffusion of ions or molecules through a gel containing a solution may take place in two ways: (1) through the liquid phase in the ordinary way, and (2) by surface diffusion if the soluble material is subject to adsorption by the solid phase. Surface diffusion is what the term implies—a migration of adsorbed ions from one part of the internal surface to another part. The movement takes place in response to a gradient in surface concentration (a gradient in the degree of saturation of the surface) just as ordinary diffusion depends on the gradient in solution concentration. Therefore, the relative amounts of adsorbed lime and adsorbed alkali that reach the reaction site should depend on the relative amounts adsorbed in *the outer part* of the gel layer. Kalousek's work indicates that when excess $Ca(OH)_2$ is available, these amounts depend on the alkali concentration at the outer boundary of the layer. Fig. 4 shows how the amounts of adsorbed CaO and adsorbed Na_2O are related to the amount of alkali in solution. The same data are shown

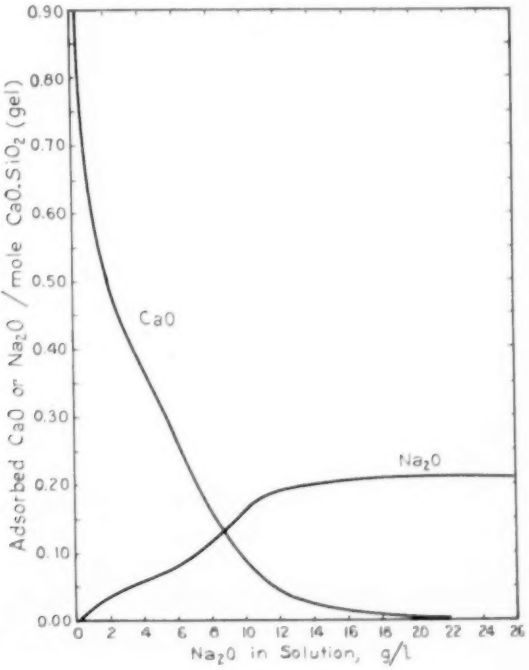


Fig. 4—Amounts of adsorbed lime and alkali in calcium silicate gel

in a different way in Fig. 5, which gives the mole ratio of adsorbed CaO to adsorbed Na_2O in relation to the concentration of Na_2O in solution.*

We can form an idea of the relative rates of diffusion from the outer boundary of the layer to the reaction site by assuming that the amounts adsorbed at the reaction site are not far from zero, that is, that most of the alkali is used in forming an alkali-silica complex, and that as the lime arrives it forms a calcium silicate with none of it left over for adsorption, thus releasing some of the alkali. (More will be said about this later.) On this assumption the relative amounts of adsorbed lime and alkali transported through the gel layer will depend mainly on the relative amounts adsorbed in the outer boundary of the layer, for it is the surface-concentration level at the outer boundary that fixes the concentration gradient. Thus, from Fig. 5 we see that if the concentration of the alkali solution in contact with the outer boundary is higher than about 20 g Na_2O per liter, the adsorbed lime is practically nil. We should therefore expect the alkali to reach the reaction site much

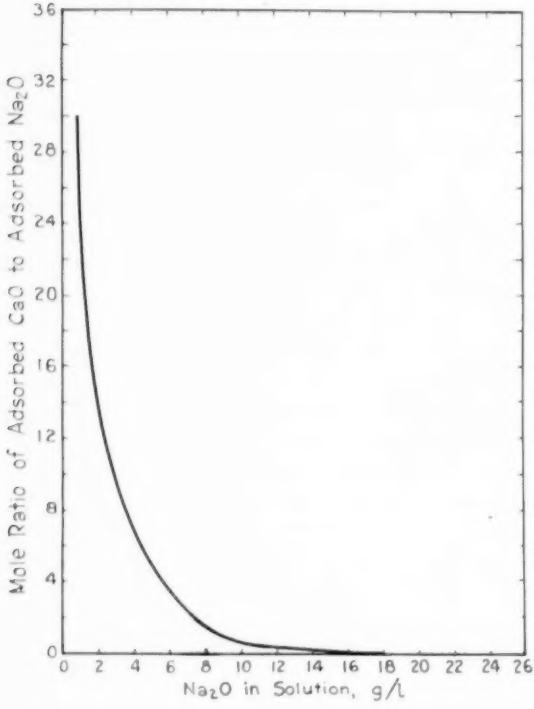


Fig. 5—Ratio of adsorbed CaO to adsorbed Na_2O in relation to Na_2O in solution

*We could have assumed that the same proportion of lime and all the alkali are held in the solid phase by solid solution rather than by adsorption. If we had done so, we would speak of the migration of lime and alkali by diffusion through the solid phase, but even so a part of this diffusion would be surface diffusion. It is simpler to assume adsorption and speak only in terms of surface diffusion. The surface diffusion would be the more rapid and might be the only diffusion of practical consequence. Indeed, this was our primary reason for assuming adsorption.

faster than the lime, and the product should be expansive.* On the other hand, if the alkali solution contained 3 g Na_2O per liter, the amount of adsorbed lime in the outer part of the layer would be about 10 times the amount of adsorbed alkali. This would indicate the possibility of the lime arriving at the reaction site at a sufficiently high rate to produce a safe product.

"Regeneration" of alkali

Although it may not at once be apparent, the above considerations indicate that the alkali in concrete may be able to react with silica as long as any unreacted silica remains. As we have seen, probably an alkali-silica complex is formed as the reaction proceeds inward, and then lime converts this product to a safe complex if the lime arrives fast enough. As the lime arrives, it reacts with the alkali-silica complex, releasing some of the alkali. Thus, some of the alkali should be "regenerated" about as postulated by Hansen.[11] Suppose, simply for purposes of illustration, that the alkali-silica complex has the formula $Na_2O \cdot SiO_2 \cdot H_2O$. The reaction with lime would then be

$$Ca(OH)_2 + Na_2O \cdot SiO_2 \cdot H_2O \rightarrow (CaO \cdot SiO_2 \cdot H_2O)_{gel} + (2-n)NaOH + n\,NaOH_{adsorbed}$$

We may think of this reaction as first forming the calcium silicate gel and thus releasing all the alkali. At the same time, the reaction product adsorbs some of the released alkali from solution. Thus, each increment of silica that combines, first with alkali and then with the lime, reduces the alkali concentration by the amount the product is able to adsorb. The amount that can be adsorbed by the product is at a maximum when the alkali concentration in solution is 20 g per liter or more. In this case, about 0.25 mole of alkali is adsorbed and about 0.75 mole is left free to react. If at the time of the reaction the alkali concentration is lower, say 5.5 g per liter, the product is able to adsorb only 0.07 mole, leaving 0.93 mole free to react with more silica (Table 2). The lower the alkali concentration at the time of the reaction, the larger the fraction of "regenerated" alkali left free to react with more silica. On this basis we conclude that some alkali will always be free to react with silica if lime can reach the reaction site. Whether complete reaction of the silica occurs must then depend on whether the level of alkali concentration remains high enough to produce such reaction. Also it depends on whether enough lime is available for a complete reaction.

Silica diffusion

Up to this point we have considered only the diffusion of lime and alkali into an opal particle. This alone cannot account for a progressive reaction. For a reaction to proceed, about half of the silica that reacts must diffuse out of the opal particle. The reason for this is that the amount of space required by the lime-alkali-silica complex is probably about twice as much as the space within the boundaries of the opal particle. This estimate is based on the similarity between the lime-alkali-silica complex and cement gel.

*This may be the circumstance that produces the "special hindrance" to lime referred to in Part 1. That is, the alkali concentration regulates the ability of the layer to transmit lime; when the alkali concentration is high, the layer is practically impermeable to lime.

Not only is the absolute volume of the solids in the complex greater than the absolute volume of the silica plus the pores in the opal particle, but also, since the complex is a porous material, room must be provided for the pores characteristic of the complex. The amount of lime and alkali that must diffuse into the particle to effect complete reaction of the particle is probably about half the amount involved in the total reaction. The rest of the reaction takes place with the soluble silica that diffuses out of the particle.

The diffusion of silica through a solution containing lime without combining with lime and precipitating until the silica reaches the region beyond the reaction layer may at first seem impossible. However, it is not only possible but apparently the process has its counterpart in the normal hydration of portland cement. Essentially the same considerations as have been advanced above apply also in the case of cement hydration. Part of the cement particle must dissolve and diffuse through the cement gel already formed without precipitating hydration products along the way. We believe we have a plausible explanation for the process of cement hydration that could apply also in the present case. The explanation, based on the peculiarities of gel formation, is too lengthy to present here. However, another possible explanation applicable to the present case may be advanced. While reaction with the opal is in progress, the equilibrium silica concentration will vary along the diffusion path according to the variation in composition of the solid phase. It is probable that the equilibrium concentration of the silica at the inner end of the diffusion path is much higher than at the outer end. If so, silica could diffuse to the outside without significantly exceeding the equilibrium solubility at any point along the way. Room for the reaction product outside the confines of the original opal particle is to be found in the originally water-filled spaces of the cement paste which fill only gradually and incompletely with cement gel.

The rate-regulating step of the reaction

According to our hypothesis we are dealing with a complex problem in diffusion leading under some conditions to an expansive reaction and under others to a safe reaction. For a safe reaction, lime must be delivered to the reaction site fast enough to react with about half the silica dissolved by the alkali. This in turn requires that the lime concentration at the outer boundary of the reaction rim be high enough at all times. This requirement involves a movement of lime and alkali from the cement paste toward the boundaries of the reactive particles. Thus, our hypothesis designates the following diffusion processes:

(1) Diffusion of alkali from the surrounding concrete to the boundary of the reactive particle

(2) Diffusion of lime to the boundary of the reactive particle

(3) Diffusion of alkali through the reacted rim of the opal particle, partly by diffusion through the interstitial liquid phase and partly by surface diffusion

(4) Diffusion of lime through the reacted rim of the opal particle, mainly by surface diffusion

(5) Diffusion of dissolved silica through the reacted rim to a place in the concrete where calcium silicate can be precipitated

Which of these diffusion processes will control the rate of reaction is not wholly clear. Our interest is, however, not so much in the rapidity with which the opal is enabled to react, as in the circumstances that may prevent the lime reaction from keeping up with the alkali reaction, with consequent development of expansive effects. When the lime reaction falls behind, this must evidently be because process (2) or (4) fails to maintain a sufficiently high rate relative to the other diffusion processes. Indeed it must evidently always mean failure of (4), but such failure may be conditioned upon failure of (2).

In the applications of the hypothesis that are made in the latter part of this paper it is tacitly assumed that diffusion process (2) is generally adequate, *i.e.*, that at any given point the amount of lime in solution will be as high as the alkali concentration will allow it to be. The assumption was adopted as a working hypothesis on the basis of the following considerations: (a) crystalline calcium hydroxide is ordinarily well distributed throughout the cement gel, and (b) this excess calcium hydroxide may dissolve, be adsorbed, and then be transferred to reaction sites by surface diffusion (on the cement gel) as well as by ordinary diffusion. There are strong indications that cement gel does hold adsorbed lime that would facilitate the transfer. Thus, in a sense, a reactive particle may draw needed lime directly from the adjacent cement gel.*

In view of the assumption made above that silica can diffuse out through the reaction layer during the progress of the safe reaction, one may wonder why the silica should become less able to migrate during the progress of an unsafe reaction, thus giving rise to osmotic pressure. Of course, until the water taken up becomes sufficient to form a sol or solution we are concerned only with the swelling of a solid phase, and the question does not arise. If an alkali silicate sol or solution is formed, the state of the silica in solution depends upon the concentration and the pH. It is quite likely that these conditions are sufficiently different from those existing during the safe reaction to permit the silicate ions to associate or form complexes and thus become too large to migrate readily. On the other hand, at the pH and silica concentrations represented by the Kalousek data of Table 2, that are applicable to cement pastes during the progress of the safe reaction, it is extremely unlikely that the silicate ions are associated.

All necessary elements of the hypothesis have now been developed; for a concise review of its content refer to the general summary, p. 809.

APPLICATION OF HYPOTHESIS TO EXPERIMENTAL DATA

Limitations of existing data

A complete check of the hypothesis would require new, and probably difficult, experimental work. The hypothesis deals mainly with the nature

*A different hypothesis results when it is assumed that step (2) rather than step (4) is the limiting one. Work on this alternative hypothesis is now under way elsewhere in this laboratory.

of the products—whether or not they contain enough lime to be nonswelling gels. Consequently, data on the amount of reaction that has taken place at a given time, and on the composition of the products formed under different conditions, and at different stages of the reaction are needed.

Existing data do not give such information, but they do show what combinations produced expansive reaction within the period of observation. Where expansion occurs, we may assume that an expansive gel was produced at one stage or another of the reaction, and we may be able to estimate when the expansive product was first present in significant amount. But when a combination is such that it *just fails* to expand within the period of observation, we cannot correctly assume that no gel of the swelling type was produced. As Vivian has shown,[12] swelling results from water being drawn from the surrounding concrete into the reacted particle. This extraction of water should produce a shrinkage that offsets some of the volume increase produced by swelling at the reaction sites. Consequently, the limit for no expansion may not coincide exactly with the limit for no reaction of the expansive type. Thus, it is only an approximation to take the limit for nonswelling as the limit for no production of swelling gel. Vivian[13] found that even when expansion is not apparent, the presence of internal pressures may show up as loss of strength.

Even though existing data have these faults (from the standpoint of checking our hypothesis) we find that the data on specimens made with opal particles do give a degree of confirmation of the hypothesis. We present an analysis of these data partly for the support of the hypothesis they afford, and partly because the analysis brings out aspects of the hypothesis of practical interest.

Analytical relationships

In applying the hypothesis stated in the preceding pages to experimental data, we find it advantageous to deal with the distribution of the alkali between the liquid and solid phase as given by Kalousek's Table 2. This gives us a reasonable indication of what complex the initial reaction could produce. Also, it indicates what the final product will be if the reaction is completed and if excess lime remains. At intermediate stages, the equilibrium relationships are no doubt not entirely reliable guides, but, as will be seen, they do seem to aid in arriving at a reasonable interpretation of experimental results.

The analytical relationships to be used are given below:

Let

Na_2O = total alkali oxide in the system, g of Na_2O or equivalent*
x = Na_2O in solution, g
y = Na_2O adsorbed by the gel, g
SiO_2 = total reactive silica in the system, g
S_r = weight of reacted silica in the gel, g

*Potassium oxide, K_2O, is calculated as the chemically equivalent amount of sodium oxide, Na_2O, by multiplying by the ratio of molecular weights, 0.66.

w_e = total evaporable water, g
c = weight of cement, g
m = fraction of total silica reacted = S_r/SiO_2

Then

$$x + y = Na_2O \quad (1)$$

and

$$\frac{x}{w_e}\frac{w_e}{c} + \frac{y}{S_r}\frac{S_r}{c} = \frac{Na_2O}{c} \quad (2)$$

On plotting the data of Table 2, we found that over the range of alkali concentrations found in hardened pastes the relationship between the amount of alkali in the solid phase and that in solution could be represented by the following equation:

$$\frac{y}{S_r} = 15\frac{x}{w_e} \quad (3)$$

Then, from Eq. (2) and (3) we obtain

$$\frac{Na_2O}{c} = \frac{x}{w_e}\left(\frac{w_e}{c} + 15\frac{S_r}{c}\right) \quad (4)$$

If at any given time a fraction, m, of the silica has reacted, the solution that would be in equilibrium with the product would have a composition satisfying Eq. (5), obtained by substituting $mSiO_2$ for S_r in Eq. (4) and solving for x/w_e.

$$\frac{x}{w_e} = \frac{\dfrac{Na_2O}{c}}{\dfrac{w_e}{c} + 15m\dfrac{SiO_2}{c}} \quad (5)$$

Let us now consider the data of Table 1 representing combinations of alkali and opal that reacted safely. Using Eq. (5) we calculated the amount of alkali in solution at various stages of the reaction. In making this calculation we are in effect assuming that the reaction proceeds so slowly that conditions are close to equilibrium at all times. Equilibrium could exist at the end of the reaction. At intermediate stages, the calculations are treated as useful approximations.

Having calculated the amount of alkali in solution, we can determine the amount of adsorbed lime that would be in the solid phase at equilibrium by making further use of Kalousek's data. A sample calculation for the third item of Table 1 at a particular stage of reaction, namely, $m = 0.5$, follows:

Given:

$Na_2O/c = 0.005$; $w_e/c = 0.34$; $SiO_2/c = 0.09$ (It is assumed here that the opal is 90 percent reactive silica.)

$m = 0.5$ (assumed)

Then, by Eq. (5),

$$\frac{x}{w_e} = \frac{0.005}{0.34 + 15 \times 0.5 \times 0.09} = 0.0048, \text{ or } 4.8 \text{ g per liter}$$

From Fig. 4 we see that the adsorbed lime is 0.32 mole per mole of silica;

from Fig. 5, the ratio of adsorbed lime to adsorbed Na_2O is 5.* The indicated complex is $1.32CaO \cdot 0.064Na_2O \cdot 1.0\,SiO_2$.

Reactions with low-alkali cements

The results of such calculations for the second and third items of Table 1 are given in Fig. 6. It should be recalled that these combinations represent the maximum amounts of alkali that could be used with the respective amounts of reactive silica without producing expansion. The first item was not used because the uncertainty regarding the safe amount of alkali was too great to serve our purpose. The curves give the equilibrium alkali concentrations from start to finish of a reaction that would consume all the silica. Notice that the caustic solutions remaining at the end of the reaction should contain about 2.5 g of Na_2O per liter. This corresponds to an 0.08 normal sodium hydroxide solution, which is certainly strong enough to attack opaline silica. Therefore, as was stated earlier, we can conclude that the alkali in these mixes made with low-alkali cements would attack the silica and that the lack of expansion could not have been due to lack of reaction. Furthermore, we must conclude that if the 18-month period was sufficient for it, all the silica must have reacted, for at no time could there have been a lack of free alkali.

These calculated alkali concentrations enable us to calculate the amount of adsorbed lime in the complex. Fig. 7 gives the calculated ratio of adsorbed

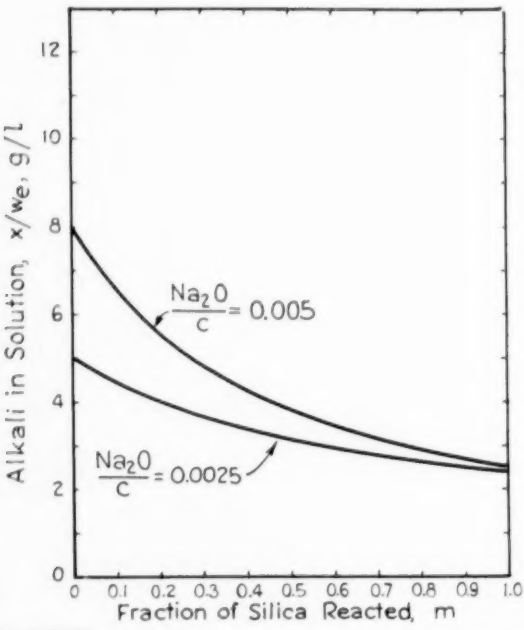


Fig. 6—Computed amounts of alkali in solution for two safe combinations at given fraction of total reaction

*Eq. (5) and this example illustrate the principle, but they do not give the method exactly. The actual method involves a correction, which is described in the appendix.

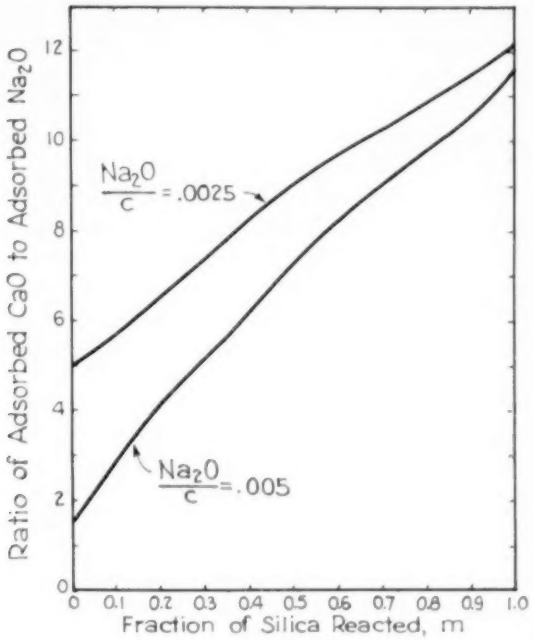


Fig. 7—Computed ratio of adsorbed CaO to adsorbed Na_2O for given fractions of total safe reactions

lime to adsorbed alkali in the reaction product for all stages of the reaction. Notice that for the cement higher in alkali the calculated amount of lime in the product is initially only a little higher than the amount of adsorbed alkali. According to our hypothesis, the reaction proceeded safely because the amount of adsorbed lime relative to alkali was able to increase immediately and at a sufficiently high rate.

The calculations do not give us the real ratios of adsorbed lime to adsorbed alkali for the early stages of the reaction, as has already been noted. But whatever actual ratio of lime to alkali the computed ratio corresponds to, it apparently was high enough (and possibly more than high enough) to assure safe reactions in the initial stages of the reaction with the higher of the two alkali contents. Certainly if the ratios were high enough for the cement containing 0.5 percent alkali, they were more than high enough for the lower alkali content.

At the final stage of the reaction the alkali concentrations (Fig. 6) and the corresponding ratios of adsorbed lime to adsorbed alkali (Fig. 7) are nearly the same for both combinations. The adsorbed lime had finally to exceed the alkali by about 12 to 1 to assure a safe final reaction.

Since each of the two combinations represents the maximum amount of alkali that could be used without producing expansion, a slightly higher alkali content would give expansion in either case. The question is, at what stage of the reaction would such expansion begin? We observe that for the

two just-safe combinations dealt with in Fig. 6 and 7 the initial conditions were much more favorable in one case than the other and that the final conditions were practically the same. It seems, therefore, that if the alkali content of each was increased slightly, the resulting expansions would be due to the change in the final condition and would not begin until the final stage of reaction was approached.

If the conclusion just reached is true, we should find expansive combinations with low-alkali cements to begin expanding at a relatively late date. A combination that might appear just safe at one time of observation might be found expansive at a later time. Woolf's data, which are the basis for Fig. 6 and 7 and the above discussion, do not at first appear to support this aspect of our hypothesis. The combinations that appeared to be just safe at 18 months also appeared to be just safe at the end of the first month. This gives the impression that Woolf's expansive combinations with low-alkali cements begin to expand at an early rather than late stage of the process. However, when Woolf's data are considered along with those from other sources, it seems likely that a relatively advanced stage of reaction was actually attained within a month. (The specimens were stored at 100 F.) Data obtained by Vivian[14] using similar combinations cured at room temperature do show the delay expected. Also, Stanton[5] using different materials observed such a delay even when curing at an elevated temperature. For example, his data on Saticoy sand (see Fig. 2) show no expansion up to 0.5 percent alkali at 3 months but at 12 months the limit for no expansion appears to be lower—0.45 percent. Evidently, reactive minerals differ in the degree to which their reactions are accelerated by a temperature rise.

Expansion due to deficiency of reactive silica

The solid curves in Fig. 8 pertain to a combination of materials that expanded. The amount of alkali in the cement is the same as for one of the two cements of Fig. 6, but the amount of reactive silica is only half as much. The calculated curves for the corresponding combination from Fig. 6 that reacted safely are indicated by broken lines, for comparison. Fig. 8 emphasizes that a deficiency of silica has the same significance as an excess of alkali. Notice that for the solid curve the final concentration of alkali is relatively high (3.8 as compared with 2.5 g per liter) and the final ratio of adsorbed lime to alkali, relatively low.

Effect of particle size

If the reaction with a low-alkali cement reaches a critical stage only in the final stages of the reaction, it follows that the maximum size of the reactive particles must be considered in determining the conditions for a safe reaction. According to our hypothesis, the rate of lime diffusion through the reacted layer depends on the surface concentrations of adsorbed lime and alkali in the outer part of the reacted layer and on the thickness of the layer through which the surface diffusion occurs. For a given rate of delivery of lime to the reaction site, the lime concentration at the outer boundary

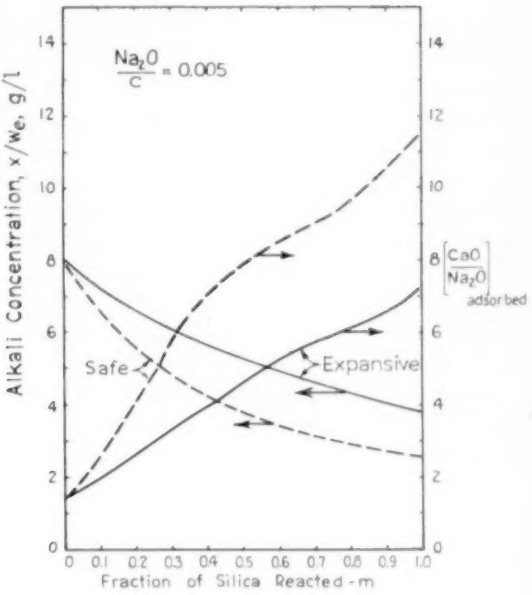

$\frac{Na_2O}{C}$ = 0.005
Alkali Concentration, x/we, g/l
$\left[\frac{CaO}{Na_2O}\right]$ adsorbed
Safe
Expansive
Fraction of Silica Reacted - m
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0 2 4 6 8 10 12 14

Fig. 8—Computed alkali in solution and ratio of adsorbed CaO to adsorbed Na_2O for safe and expansive reactions (SiO_2/c = 0.090 for safe reactions; 0.045 for expansive reactions). Note: Horizontal arrows indicate ordinate scale to read)

of the layer must be higher the thicker the layer. If a reaction has proceeded safely up to a given depth of penetration into an opal particle, the lime concentration must have been high enough at all times to deliver lime to the inside of the layer at the proper rate. If the lime concentration in the outside part is *just sufficient* to maintain the required rate at a given depth of penetration, it must increase thereafter, and at an adequate rate, if further safe penetration is to result. It does increase with the progress of the reaction, for it increases as alkali concentration decreases. But whether or not it increases at the proper rate depends upon the particular circumstances.

As already noted, Fig. 7 indicates that when using 16–30-mesh opal particles the final stage of reaction will not occur safely unless the final product contains at least 12 times as much adsorbed lime as adsorbed alkali, which means that the final alkali concentration must not exceed 2.5 g per liter. Had the particles been larger, the lime would have been required to diffuse through a greater distance and therefore the final alkali concentrations would have had to be lower than 2.5 g per liter to permit the needed build-up of adsorbed lime in the outer part of the reacted layer.

If 2.5 g per liter represents the safe final alkali concentration for 16–30-mesh opal particles, it is also a safe concentration for smaller particles. In general our hypothesis indicates that the smaller the size of the particles the higher will be the just-safe final alkali concentration, provided that the critical stage of the reaction occurs near the final stage. The indications from the hypothesis when the critical stage occurs early in the reaction, as

seems to be the case with high-alkali cements, are developed in the next section.

Reactions with high-alkali cements

Let us now consider reactions between opal particles of the kind discussed above in a mix made with a high-alkali cement. The data given in Fig. 1 show that with $Na_2O/c = 0.01$, the least quantity of 18- to 52-mesh opal particles that would give a safe reaction during 224 days (7½ months) was 80 g of opal per g of Na_2O. With low-alkali cements, safe reactions were obtained with mixes containing from 15 to 22 g of opal per g of alkali, depending on the alkali content. Why was about four times as much silica per g of alkali needed for a safe reaction in mixes made with only twice as much alkali?

An explanation in terms of our hypothesis can be given with the aid of Fig. 9, in which the results of calculations for the two high-alkali cements are plotted. The calculations were made for a combination that was just safe with the high-alkali cement and for a combination that was expansive. From the various expansive combinations (see Fig. 1), we chose one which, if it had reacted safely, would have ended in a caustic solution having the same strength as that produced by the just-safe reactions with the low-alkali cement, namely, 2.5 g Na_2O per liter. The explanation can be developed most clearly from Table 3 derived from Fig. 9 and Fig. 6 and 7.

As shown in the final column, the first three lines of this table give calculated values for two just-safe combinations and one expansive combination with low-alkali cements. The fourth and fifth lines represent a safe and an expansive combination with the high-alkali cement.

The just-safe combinations with the low-alkali cements show practically equal final ratios of adsorbed lime to adsorbed alkali (column 6) but different initial ratios (column 5), as was pointed out in the preceding section. The expansive combination (line 3) has the same initial ratio (column 5) as the safe combination (line 2), but a substantially lower final ratio.

Line 4, representing an expansive combination with a high-alkali cement, shows the same final ratio as the line 1 and line 2 combinations; but the initial condition is different. This initial ratio of adsorbed lime to adsorbed alkali, not given in the table, is shown by Fig. 9 to have been only 0.3:1.

TABLE 3—CALCULATED CONDITIONS FOR JUST-SAFE AND EXPANSIVE COMBINATIONS WITH BOTH LOW- AND HIGH-ALKALI CEMENTS

Alkali content of cement, percent	SiO_2/Na_2O	Vol. of silica reacted, percent of total	Radial penetration, percent	Ratio of adsorbed CaO to adsorbed Na_2O at indicated penetration	Final ratio adsorbed CaO to adsorbed Na_2O	Result
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.25	17.5	0	0	5:1	12.2	Just safe
0.50	21.3	0	0	1.4:1	11.6	Just safe
0.50	11.0	0	0	1.4:1	7.3	Expansive
1.00	24.3	14	5	1.4:1	11.6	Expansive
1.00	72.0	2.5	1	1.4:1	>30.0	Just safe

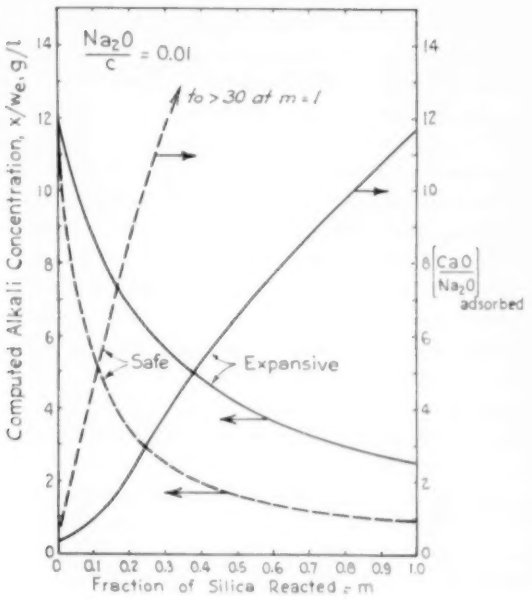

$\frac{Na_2O}{C}$ = 0.01
to > 30 at m = 1
Safe
Expansive
$\left[\frac{CaO}{Na_2O}\right]$ adsorbed
Computed Alkali Concentration, x/we. g/l
Fraction of Silica Reacted = m
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0 2 4 6 8 10 12 14

Fig. 9—Computed alkali concentration and ratio of adsorbed CaO to Na_2O in combinations with a high-alkali cement (SiO_2/Na_2O = 72 for safe reactions; 24.3 for expansive reactions). Note: Horizontal arrows indicate ordinate scale to read

The ratio did not reach the value 1.4: 1, which was the initial ratio for the 0.5 percent alkali of line 2, until 14 percent of the silica had reacted (see table). This amount of reaction corresponds to a radial penetration of 5 percent (column 4), assuming even penetration of a spherical particle of reactive silica.

Thus, on comparing lines 4 and 2 we see that for line 4 the conditions for lime diffusion were the less favorable at the start, and on comparing corresponding curves in Fig. 7 and 9 we see that they remained less favorable until the very end of the reaction. The calculations thus are in qualitative agreement with the fact that the line 4 combination was expansive and the line 2, safe. Since the conditions appear, by calculation, to have improved as the reaction progressed, and since the final condition was the same as that of a safe reaction (line 1 or line 2), it seems that the expansion must have begun at an early stage of the reaction.

The just-safe combination with the high-alkali cement represented in line 5 also started under less favorable conditions than the line 2 just-safe combination. The conditions remained less favorable until 2.5 percent of the silica had reacted, corresponding to a 1 percent radial penetration. Thereafter, the condition for lime diffusion for line 5 was more favorable than for line 2 at corresponding stages of the reaction. For example, at 30 percent of the total reaction, representing a radial penetration of 11 percent, the ratio of adsorbed lime to adsorbed alkali for line 5 is 12: 1 and for line 2, 6: 1. The corresponding caustic strengths are 2.5 g per liter and 6 g per liter, respectively. Hence, it seems that the safeness of this reaction as compared

with that of line 4 was due to the difference in radial penetration at the early stages of the reaction.

As already pointed out, we do not believe that calculations such as those shown in Table 3 give accurate comparisons, but they do serve to illustrate our hypothesis that with high-alkali cements expansion will occur unless the reactive silica is able to reduce the caustic strength to a safe level with only a slight penetration of the reactive silica particles. With the line 5 combination a reaction producing a 1 percent radial penetration reduced the calculated caustic strength to 7 g per liter (Fig. 9), which is a little less than the initial caustic strength with the 0.5 percent alkali cement. The combination in line 4 required a calculated 5 percent radial penetration of the particles to reduce the alkali concentration to this level. Evidently, a radial penetration of 5 percent in particles of the size used could not be effected safely at the ratios of adsorbed lime to adsorbed alkali that could exist at concentrations above 7 g per liter, that is, less than 1.4 adsorbed CaO to 1 adsorbed Na_2O.

That the layer formed safely at high-alkali concentrations must be very thin is shown also by Vivian's experiments with opal particles of various sizes.[4] He found that 200–300-mesh particles and all larger sizes could react safely with a 0.9 percent alkali cement, only if the amount of opal was somewhere between 60 and 70 g per g of alkali. With pulverized opal, all of which passed 300 mesh, any amount of silica down to the least amount used (1 percent) was found to react safely. This indicates that the thickness of the safe layer that can form in the relatively strong caustic solution is less than 25 microns, the radius of a 300-mesh particle, approximately. Probably the safe thickness is substantially less than that, for, as we have seen, the just-safe combination with the high-alkali cement shows a reduction of caustic strength to the low-alkali level after a 1 percent radial penetration of 18–52-mesh particles. The largest particle that can pass an 18-mesh sieve (British) has a diameter of about 800 microns. Hence, 1 percent penetration corresponds to only 8 microns. Presumably, in the experiments with opal passing 300 mesh, the very low percentages were successful because the maximum size particles were few in number, and the higher percentages were successful because of the rapid reduction in alkali concentration caused by the reaction of the many extremely fine particles.

The relatively high proportion of silica particles needed to give a safe reaction with 1 percent alkali (72 percent of the weight of the alkali) was dictated by the necessity of keeping the radial penetration below a safe limit while the alkali concentration remained relatively high. The necessary amount of silica was learned by experiment, as is shown in Fig. 1. During the initial stage, when the alkali concentration was dangerously high, it is possible that the unsafe reaction outstrips the safe reaction. But if the reaction layer is thin, the reduction in concentration effected by this initial reaction may then permit the safe reaction to catch up. It seems possible that during the interval before the safe reaction takes over, some expansive

complex may be produced, but it does not have time to imbibe water and swell before the safe reaction overtakes it and converts it to the safe condition.

The detailed analysis given above leads to the conclusion that with high-alkali cements expansive reaction is liable to begin near the start of the reaction and with low-alkali cements it is liable to begin near the end. To prevent expansive reaction with a high-alkali cement, the total surface area presented by the silica must be high enough to keep the radial penetration low until the safe reaction can begin to control. This it can do if the layer is still thin when the alkali concentration has been strongly reduced. With a lower-alkali cement, where the critical stage is reached near the end of the reaction, the amount of silica must be sufficient for the safe reaction to *maintain* control. For it to maintain control, the number of silica particles must be such that, with the progress of the reaction toward the centers of the particles, a satisfactory gradient of adsorbed lime is still maintained. That is, the lowering of the alkali concentration caused by the progress of the reaction brings about concomitant increase in the adsorbed lime of the outer layer which, if sufficiently great, can maintain the adsorbed lime gradient in spite of the increasing thickness of reaction layer. Therefore, for particles of a given kind and for a given amount of alkali, the amount of silica required for a safe combination depends on the size of the silica particles.

INTERPRETATION OF RESULTS FOUND IN PRACTICE

Why cements with less than 0.6 percent alkali generally are safe

Our hypothesis, together with data on the alkali concentrations in hardened cement paste, explains why low-alkali cements usually show no expansion in the mortar bar test. The explanation comes from a consideration of the changes in alkali concentration (and the consequent changes in the ratio of adsorbed lime to adsorbed alkali in the outer parts of the reacted layers) that occur during the course of a given reaction. Conditions at the start are shown in Fig. 10 for cements of varied alkali content. This figure was derived from Fig. 3 of the preceding paper[1] and Fig. 5 of this one. It shows the computed ratio of adsorbed lime to adsorbed alkali in the first-formed siliceous reaction product in relation to alkali content of cement, for two different water-cement ratios. Since in mortar bar tests the water-cement ratio is usually near 0.5 by weight, we will focus attention on the lower curve.

A point on the lower curve at an alkali content of 0.4 percent, for example, shows that the ratio of adsorbed lime to adsorbed alkali would be 1.8 in a product that would be at equilibrium with the solution initially present in a paste having the water ratio indicated. It therefore indicates the ratio of adsorbed lime to adsorbed alkali in the reaction product before the amount of reaction is enough to change the initial concentration appreciably.

(Before proceeding we must remark that again we are indulging in over-simplification in that we have supposed that all the alkali has gone into solution before any reaction with silica takes place. Also, we have assumed that the cement gel has already adsorbed the amount of alkali that can be

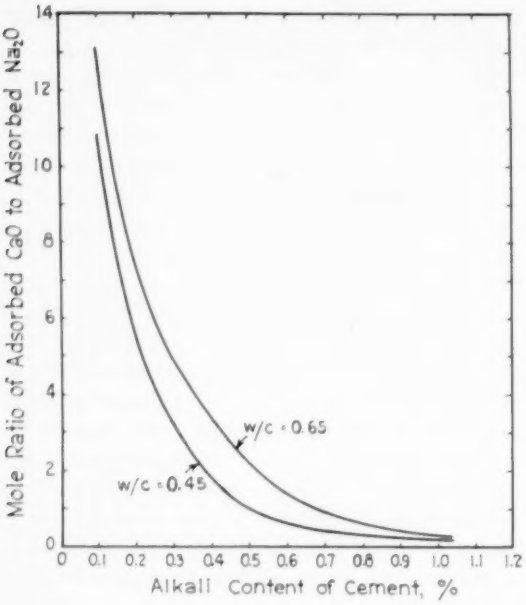


Fig. 10—Ratio of adsorbed CaO to adsorbed Na_2O in the complex that would be in equilibrium with solutions found in mature pastes

held in a mature paste. Even though this cannot agree exactly with fact, the considerations being presented lead to a reasonable explanation of the question before us.)

As the reaction proceeds from this point, the alkali concentration decreases from its initial value. Each increment of reaction thus occurs in a weaker alkali solution than that for the preceding increment. It is apparent that the ratio of adsorbed lime to adsorbed alkali would begin to rise immediately after reaction starts. The previous considerations indicate that the reaction starts under safe conditions. Whether it continues under safe conditions will depend upon the rate at which the ratio of adsorbed lime to adsorbed alkali rises as the particles are penetrated by the reaction. We have seen that a sufficient rate of increase in the ratio can be maintained if there is a sufficient quantity of 16–30-mesh opal particles. If some of the particles are smaller, then, for maintenance of a given rate of rise of the ratio with respect to particle penetration, less opal is necessary. From results obtained in the field, it appears that the reactive mineral, when present, is usually there in sufficient amount and of sufficient fineness to keep a 0.6 percent alkali oxide content (of cement) under control. A small amount of quite fine particles goes a long way in this direction.

Now, consider the point on the lower curve representing an alkali content of 1 percent. At the start the amount of adsorbed lime is practically nil. As the reaction proceeds the caustic strength falls, but, as shown in Fig. 5, the ratio of adsorbed lime to adsorbed alkali does not begin to rise rapidly

until the ratio exceeds about unity. In Fig. 10 we see that for a water-cement ratio of about 0.5 the ratio does not reach unity until the caustic strength has been reduced to the point where it would have been at the start with cement containing between 0.5 and 0.6 percent alkali. Thus, the reaction begins under unfavorable conditions. A safe reaction can occur if the reactive mineral in the aggregate presents enough surface to reduce the caustic strength sufficiently to produce favorable conditions before more than a shallow surface reaction of the larger particles has occurred. If the amount of surface is too small for this, the unsafe reaction will continue and expansion will occur. Since many tests have shown expansive reactions at alkali contents above 0.6 percent, we infer that under these conditions the surface exposure of the reactive mineral is usually insufficient.

The indications from Fig. 10 are that if the mortar bars were made with leaner mixes, giving water-cement ratios higher than 0.5, the indicated safe alkali content would be a little higher than 0.6 percent.

Function of pozzolanic additions

The foregoing suggests an explanation for the effectiveness of pozzolanic additions in preventing expansive reactions with high-alkali cements. When we add a powdered reactive mineral we thereby expose a large area of reactive surface to the solution in addition to that of the coarser reactive particles already present. For a given amount of reaction and thus a given lowering of the alkali concentration, the penetration of the coarser particles is thus greatly reduced. If the amount of fine particles is adequate, the initial reaction decreases the alkali concentration to what it would be for a safe reaction with a low-alkali cement and the coarser particles may then continue their reaction safely. Evidently, the addition must be such as to reduce the alkali concentration to about what it would be with the cement containing 0.6 percent alkali. The test data we have examined indicate that for several different pozzolanic materials the amount required in each case was that giving about 20 g of reactive silica per g of alkali in excess of 0.5 percent. Probably the quantity of pozzolanic material needed would be dependent on the fineness.

In Fig. 3 it will be noted that adding pulverized opaline chert did not have the same effect in all cases. With the cement highest in alkali, amounts up to 6 percent increased the amount of expansion, whereas with the other cements the same addition lowered expansion. Other similar results have been reported. Such results show that if a pozzolan is added to an expansive combination, in an amount too small to meet the requirements for a safe reaction, the expansion may be increased or decreased, depending on factors not dealt with in our hypothesis. It is therefore advisable to add an amount that will surely produce a safe reaction.

Significance of hypothesis with respect to field surveys

This hypothesis, supported by the data of Davis and Woolf, indicates that expansive reaction with low-alkali cements is liable to occur if the

particle size of the reactive component of the aggregate is relatively large. A coarse aggregate containing opal used with a sand free from reactive mineral might develop expansion in the presence of a small amount of alkali. Or a sand containing reactive mineral and unusually deficient in fines might cause trouble with low-alkali cement.

When conditions such as these exist in a given geographical area, it does not seem likely that field-survey data analyzed statistically would reveal it. For example, if the reactive mineral in a given area occurs only in particle sizes larger than 50 mesh, expansion might occur at say 0.4 percent alkali when the percentage of reactive mineral is below a certain value and the reaction would be safe when the percentage of reactive mineral is above that value. If this percentage happened to be the average for the area surveyed, the survey would reveal as many cases of nonexpansion as expansion with 0.4 percent alkali. The alkali would thus appear statistically to have no connection with the phenomenon. Yet it could still be true that every case of expansion was due to the alkali-aggregate reaction.

The possibility that reactive aggregates unusually deficient in fines may cause trouble when used with low-alkali cements could easily be checked by suitable laboratory tests and indeed may already have been checked by some of the tests of Conrow.[15]

EFFECT OF DRYING

Vivian[13] has shown that the alkali in concrete can attack reactive silica even when the concrete is apparently dry. In structures built in arid regions significant reductions in evaporable water are probable. Therefore, if a reaction is to occur safely, it must be able to do so in the presence of a smaller amount of water than that required to saturate the concrete.

In the laboratory mortar bar test the specimens are kept close to saturation by storing them in closed containers over water. However, other tests have been devised in which specimens are subject to drying and wetting cycles, the drying being aided by elevated temperatures. From our hypothesis we would predict that a period of drying occurring before the silica reaction has become completed might turn a safe reaction into a detrimental one, especially if the concrete is later wetted. Consider, for example, the upper curve of Fig. 6 at the point where 47 percent of the silica is indicated to have been reacted ($m = 0.47$). The strength of the alkali solution has been reduced from its initial value of 8 g per liter to a calculated 4 g per liter and up to this point the reaction has proceeded safely. Now if half the evaporable water should be quickly removed, the alkali concentration would be increased by a factor of about 2, that is, from 4 to 8 g per liter. If such an alkali concentration were produced at this stage of the reaction, it would probably prevent the continued diffusion of lime through the reaction layer at the rate required for a safe reaction. The reactions would therefore produce an expansive alkali-silica complex. As long as the specimen remained in the dry state, the unsafe reaction might not be evidenced by growth of the

specimen in the usual way. However, when the specimen is rewetted, it may show rapid expansion.

Consider the effect of oven drying. When a specimen is placed in an oven at 130 F, for example, it can retain about the same amount of water as when dried at room temperature at 10 percent humidity. We estimate that if the evaporable water content at saturation is 34 percent of the weight of the cement, after the specimen has been dried at 130 F its evaporable water content is about 6 percent. Under this condition the alkali concentration when using a cement with only 0.2 percent alkali could be as high as or higher than the concentration in a saturated sample made with a cement containing 1 percent alkali. It does not seem surprising therefore that Conrow observed rapid expansion with various cements having alkali contents ranging from 0.2 to 0.73 percent after the specimens had been in an oven at 130 F for 7 days and then stored in water. Even though Conrow's test conditions were such as to permit leaching of a considerable part of the alkali, only a little alkali was required to account for the attack under the conditions described. The fact that he could prevent the expansion by a suitable use of pozzolan is in harmony with this explanation.

GENERAL SUMMARY

1. Published experimental data on specimens containing reactive aggregate show that the cement can contain some alkali without developing expansion from the alkali-aggregate reaction. The safe amount of alkali depends on the amount of reactive mineral and the size of the reactive particles.

2. A hypothesis concerning reactions with opal has been developed on the basis of the following known facts. Alkali hydroxide can attack opal, forming alkali silicate. Hydration of portland cement saturates the liquid phase with calcium hydroxide. This calcium hydroxide can react with alkali silicate to precipitate lime-alkali-silica complexes. The composition of the complex is dependent on the alkali concentration in solution.

3. The hypothesis may be stated as follows:

(a) The initial reaction produces a thin layer of lime-alkali-silica complex (a nonswelling gel) that separates the unreacted silica from the lime and alkali in the concrete surrounding the particle. The layer of gel contains adsorbed lime and adsorbed alkali that are available to the unreacted silica.

(b) The relative amounts of adsorbed lime and adsorbed alkali in the reacted layer of opal are controlled by the alkali content of the solution external to the layer.

(c) Adsorbed lime and adsorbed alkali diffuse through the layer to react with silica. If the ratio of adsorbed lime to adsorbed alkali in the outer part of the layer is kept high enough, lime is delivered to the reaction site fast enough to produce a nonswelling gel. This means that the alkali concentration external to the layer must not be too high at any stage of the reaction.

(d) If the ratio of adsorbed lime to adsorbed alkali in the outer part of the layer is not kept high enough, lime cannot reach the reaction site fast enough. Thus, an alkali-silica complex, or gel, is produced which can imbibe water and swell, causing expansion of concrete.

(e) When a reaction is proceeding safely, forming lime-alkali-silica complex, the reaction of the lime releases some of the alkali that reacted initially. The "regenerated" portion of the alkali is free to continue the attack on the silica. The indications are that under many conditions the safe reaction can continue until all of the reactive silica has been converted to a lime-alkali-silica complex.

(f) For a safe reaction to proceed, a part of the silica (probably about half) must diffuse out of the particle and react with lime and alkali outside the particle.

4. Existing data are not ideal for checking the hypothesis, but they give a degree of confirmation. The data are interpreted by means of Kalousek's data on the distribution of alkali between the solid and liquid phase and the corresponding ratio of adsorbed lime to adsorbed alkali at equilibrium in the presence of excess lime. Such interpretation of data of Woolf and Davis and Vivian leads to these results:

(a) For cements containing up to about 0.6 percent alkali oxide the alkali concentration appears to be low enough for safe *initial* reaction. For safe conditions to continue, the amount of reactive silica must either be negligible or more than a certain minimum determined both by the amount of alkali and the fineness of the silica.

(b) With high-alkali cements, the *initial* conditions promote unsafe reaction with reactive silica. Rapid reversal to safe conditions is necessary and will occur if the reactive silica presents sufficient surface area. However, if the silica particles are relatively uniform and not extremely fine, the required ratio of reactive silica to alkali oxide is much higher than for low-alkali cement.

(c) The effectiveness of a good pozzolanic addition, in preventing expansion, is due to the large area of reactive surface that the pozzolan provides. Thus the high-alkali concentration produced by high-alkali cement is reduced sufficiently to permit safe reaction before the larger reactive silica particles have undergone more than shallow, surface reaction. Presumably, after that, the lime reaction can overtake the alkali reaction.

5. Cements with less than 0.6 percent alkali are usually safe in the mortar bar test, and in concretes containing reactive silica, apparently because the amount and fineness of the reactive silica usually meet the requirements mentioned in 4a.

6. Field surveys analyzed statistically might not reveal existing cases of expansion due to alkali-aggregate reaction, if the reactive mineral is coarse and low-alkali cements are involved.

7. A safe reaction might be converted to an unsafe one if the specimen is dried and resaturated before the reaction is complete.

We believe that previously established conclusions concerning the safe alkali content of concrete should be re-examined, as has already been suggested by Hanna[16] and by Woolf.[6] Also, we hope the considerations set forth in this paper will result in further laboratory studies such as seem required for an adequate test of the hypothesis.

REFERENCES

1. Powers, T. C., and Steinour, H. H., "An Interpretation of Some Published Researches on the Alkali-Aggregate Reaction. Part 1—The Chemical Reactions and Mechanism of Expansion," ACI JOURNAL, Feb. 1955, *Proc.* V. 51, pp. 497-516.

2. Vivian, H. E., "Studies in Cement-Aggregate Reaction: X. The Effect on Mortar Expansion of Amount of Reactive Component," *Bulletin* No. 256, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1950, pp. 13-20.

3. Stanton, T. E., "Expansion of Concrete through Reaction between Cement and Aggregate," *Transactions*, ASCE, V. 107, 1942, p. 54 (author's closure to discussion).

4. Vivian, H. E., "Studies in Cement-Aggregate Reaction: XIX. Effect on Mortar Expansion of the Particle Size of the Reactive Component in the Aggregate," *Australian Journal of Applied Science*, V. 2, 1951, pp. 488-494.

5. Stanton, T. E., "Studies of Use of Pozzolanas for Counteracting Excessive Concrete Expansion Resulting from Reaction between Aggregates and the Alkalis in Cement," Symposium on Use of Pozzolanic Materials in Mortars and Concretes, ASTM *Special Technical Publication* No. 99, 1950, pp. 178-201 (discussion pp. 202-203).

6. Woolf, D. O., "Reaction of Aggregate with Low-Alkali Cement," *Public Roads*, V. 27, No. 3, 1952, p. 50.

7. Davis, C. E. S., "Studies in Cement-Aggregate Reaction: XVIII. Effect of Soda Content and of Cooling Rate of Portland Cement Clinker on Its Reaction with Opal in Mortar," *Australian Journal of Applied Science*, V. 2, No. 1, 1951, pp. 123-131.

8. Vivian, H. E., "Studies in Cement-Aggregate Reaction: XIV. The Effect of Small Amounts of Reactive Component in the Aggregate on the Tensile Strength of Mortar," *Bulletin* No. 256, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1950, pp. 53-59.

9. Kalousek, G. L., "Studies of Portions of the Quaternary System Soda-Lime-Silica-Water at 25° C," *Journal of Research*, National Bureau of Standards, V. 32, 1944, pp. 285-302.

10. Vivian, H. E., "Studies in Cement-Aggregate Reaction: XV. The Reaction Products of Alkalis and Opal," *Bulletin* No. 256, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1950, pp. 60-78.

11. Hansen, W. C., "Studies Relating to the Mechanism by Which the Alkali-Aggregate Reaction Produces Expansion in Concrete," ACI JOURNAL, Jan. 1944, *Proc.*, V. 40, pp. 213-227.

12. Vivian, H. E., "Studies in Cement-Aggregate Reaction: II. The Effect of Alkali Movement in Hardened Mortar," *Bulletin* No. 229, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1947, pp. 47-54.

13. Vivian, H. E., "Studies in Cement-Aggregate Reaction. VII. The Effect of Storage Conditions on Expansion and Tensile Strength-Changes of Mortar," *Journal*, Council for Scientific and Industrial Research, Australia, V. 20, 1947, pp. 585-594.

14. Vivian, H. E., "Studies in Cement-Aggregate Reaction: XVII. Some Effects of Temperature on Mortar Expansion," (Table 2, data for 5 percent opaline rock and for 10 percent siliceous magnesian limestone at room temperature), *Australian Journal of Applied Science*, V. 2, No. 1, 1951, pp. 114-122.

15. Conrow, A. D., "Studies of Abnormal Expansion of Portland Cement Concrete," *Proceedings*, ASTM, V. 52, 1952, p. 1205.

16. Hanna, W. C., "Unfavorable Chemical Reactions of Aggregates in Concrete and a Suggested Corrective," *Proceedings*, ASTM, V. 47, 1947, p. 986.

Appendix—Details Pertaining to Calculations Such as Those Represented in Fig. 6

To calculate the amount of alkali in solution at any given stage of the reaction by means of Eq. (5), we must take into account the fact that some of the alkali is held out of solution by the cement gel. This may be done by arbitrarily assuming that the cement gel does not hold any alkali and that, instead, a certain amount of reacted silica in addition to the unreacted opal is present at the start. This hypothetical amount of silica was calculated from the following form of Eq. (4):

$$\frac{S_h}{c} = \frac{\frac{Na_2O}{c} - \frac{w_e}{c}\frac{x}{w_e}}{B\frac{x}{w_e}}$$

where S_h/c = hypothetical amount of reacted silica present before any of the opal reacted, g per g of cement.

After the reaction with opaline silica gets under way, the fraction of opaline silica reacted at any given stage of the reaction is

$$\frac{S_r}{SiO_2} = m$$

The fraction of the *total* reacted silica (opaline + hypothetical) is

$$\frac{S_r + S_h}{SiO_2 + S_h} = m' = \frac{S_r + S_h}{SiO_2'}$$

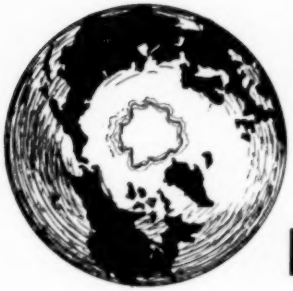
We therefore first calculated m' and the corresponding values of x/w_e, using Eq. (5).

Having obtained x/w_e versus m', we calculated x/w_e versus m by the relationship

$$m = \frac{m' - \frac{S_h}{SiO_2'}}{1 - \frac{S_h}{SiO_2'}}$$

In the calculations, the evaporable water, w_e/c, was taken as 0.35, that being appropriate for a Type I cement at $w/c = 0.5$, at a late age. The initial alkali concentration, x/w_e, was taken from Fig. 3 of Part 1. B was taken as 15 or a lower value from Table 2, according to the initial strength of the solution.

We recognize that these calculations do not conform to all the realities. We have simplified the problem by assuming that the cement completes its hydration before the reaction with the opal begins; and we have, in effect, treated the silica of the cement as though it were a lesser amount of opal (determined by Fig. 3 of Part 1). However, the calculated results serve to indicate certain qualitative aspects of a complex process and seem to us to be justified on that basis.



Letters from Readers*

BY WAY OF SYNOPSIS

Three readers contribute to the vastly growing list of words which are "derived" from the word concrete.

LEO LIBERTHSON describes the use of radioactive tracer techniques in evaluating the penetration of fluosilicate solutions into concrete with and without wetting agents.

Concrete's Etymological Offspring (LR 51-18)

This series,† which began in 1950 with a reader's amusement at the number of words (16 he thought) that were the offspring of the word concrete, has brought to light an increasing number of such words. For this fifth progress report, "A Reader" is joined by two other "researchers of the word" in adding 120 new words to the list. As of March 1 the list stands at 310 words.

—EDITOR

* * * * *

I submit the following new additions to the list of concrete's etymological offspring.

Word	Where used
Arld Crete	Arld Crete Co., Cincinnati, Ohio
Auto-Crete	(Concrete mixer) Allison Steele Co., Phoenix, Ariz.; *Better Roads*, Apr. 1954, p. 59
Cellucrete	Willson, patented 1943
Crete Seal	(Curing compound) Calbar Paint and Varnish Co., Philadelphia, Pa.
Evercrete	(Waterproofing) Evercrete, Inc., New York N. Y.; *Thomas' Register*
Fibercrete	McMillan, U. S. Patent 1907080; Kelley Island Lime and Transport Co., Cleveland, Ohio
Hamcrete	*Pit and Quarry*, May 1954, p. 156
Hydrecon	(Refractory concrete)
Koncrete	Koncrete Krushers, *Pit and Quarry*, Aug. 1954, p. 46
Metalcrete	Metalcrete Co., Cleveland, Ohio
Modern Crete	Modern Crete Production Co., Kenova, W. Va.; *Rock Products*, Aug. 1954, p. 212
National-Crete	(Cellular concrete) National Foam System, Inc., West Chester, Pa.; ACI JOURNAL, *News Letter*, May 1954, p. 10

*A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Separate prints of the entire Letters from Readers section are available at 35 cents each. Address 18263 W. McNichols Rd., Detroit 19, Mich.

†ACI JOURNAL, *Proc.* V. 47, Nov. 1950, p. 263, May 1951, p. 752; *Proc.* V. 49, Mar. 1953, pp. 682-684; *Proc.* V. 50, June 1954, pp. 894-897.

Pipecrete	Pipecrete Inc. (Manufacturers of concrete pipe)
Precrete	(Prestressed concrete manufacturer) Precrete, Inc., New York, N. Y.; *Engineering News-Record*, July 22, 1954, p. 64
Stresscrete	(Prestressed roofing block) Atlas Building Products Co., El Paso, Texas; *Contractors and Engineers*, June 1954, p. 32
Tuferete	(Waterproofing) Tuferete Co., Inc., Des Moines, Iowa; *Thomas' Register*

Here is some additional information on words previously submitted.

Word	New data
Curcrete	Add Philadelphia (Miss.) Concrete Co.
Novacrete	Also (Lightweight concrete) Garrow, U. S. Patents 1537406, 1571048, and 1583713
Precon	Also Precon Concrete Products Co., Clayton, Mo.; *Pit and Quarry*, May 1954, p. 256
Stone-crete	Add Thompson Materials Co., New York, N. Y.

A READER

* * * * *

Below are some "crete" words that I have found since the last progress report on concrete's etymological offspring.

Word	Where used
Arterete	(Concrete benches and other products) Arterete Products Co., Havertown, Pa.; *House and Garden*, May 1954, p. 59
Blokcrete	The Blokcrete Co., Ltd., Southampton, England; ACI JOURNAL, May 1954, p. 796 (item 67)
Concreteclad	MacQueen Service; *Building Supply News*, Feb. 1954, p. 732
Crete-Color	Wilbur & Williams Co., Brighton, Mass.; *Building Supply News*, Feb. 1954, p. 733
Duro-Crete	Duro-Crete Co., East Providence, R. I.; Providence telephone directory
Elasticrete	(Cellular lightweight concrete) Elastizell Corp. of America, Alpena, Mich.; *Pit and Quarry*, Oct. 1954, p. 182
Geocon	Geocon, Ltd., (formerly Gunite & Waterproofing Ltd.) Halifax-Montreal-Toronto-Dorval(Canada); *Engineering News-Record*, Oct. 28, 1954, p. 66
Grancrete	Grancrete Co., Inc., Boston, Mass.; Providence telephone directory
Greencon	(Saw for green concrete) Clipper Mfg. Co., Kansas City, Mo.; *Engineering News-Record*, July 22, 1954, p. 5
Hilcon	(Floor and wall slabs) Hills (West Bromwich), Ltd., England; *Concrete and Constructional Engineering*, June 1954, p. xix
Hyprescon	(Steel - cylinder - reinforced concrete pipe) Pressure Pipe Co. of Canada, Ltd., Montreal, Canada; *Engineering Journal*, Jan. 1955, p. 20.
Jetereter	(Concrete placing gun) Construction Machinery Sales Co., Waterloo, Iowa; *Contractors and Engineers*, Sept. 1954, p. 80
Konkreto	(Rubberized floor enamel) Murphy Paint Co.; *Hartford Times*, Sept. 16, 1954, p. 6
Laticrete	(For repairing concrete floors, lining tanks, bedding tiles, etc.) Dominion Rubber Co., Ltd., Montreal; *Engineering Journal*, Oct. 1954, p. 1364

Nomocrete	(Slide-rule attachment for proportioning simple R/C beams) Gesellschaft der Ludwig von Roll'schen Eisenwerke AG, Gerlafingen; *Schweizerische Bauzeitung*, July 31, 1954, p. 455
Pumpkret	(Concrete pump) Beton-Spritz-Maschinen GmbH., Frankfurt am Main, Germany
Qualicrete	*Pit and Quarry*, Oct. 1954, p. 120
Split-Crete	(Masonry units) Park Ave. Cement Block Co., Cranston, R. I.; Providence telephone directory
Stone-Crete	(Block and brick) Park Ave. Cement Block Co., Cranston, R. I.; Providence telephone directory
	(Concrete and masonry paint) Tamms Industries, Inc., Chicago, Ill.
Strescon	(Prefabricated prestressing units) Prestressed Concrete Corp., Kansas City, Mo., and Torrance, Calif.
Torkret	(Grout injection machine) Torkret GmbH, Berlin, Germany; *Bauingenieur*, Aug. 1954, adv. p. 3

The following words are from the 1953 edition of *Handbook of Material Trade Names*. A page-by-page search yielded a number of words not listed under the usual catagories in the directory section.

Blazecrete	(Castable refractory) Johns-Manville Sales Co., New York, N. Y.
Car-Na-Crete	(Concrete floor surfacer) Continental Car-Na-Var Corp., Brazil, Ind.
Conco	(Concrete floor paint) Aquabar Co., Philadelphia, Pa.
'Crete-Clean	(Cleaner for masonry surfaces) Wilbur & Williams Co., Brighton, Mass.; (Tradename section of handbook)
'Crete-Cleen	(See 'Crete-clean) Wilbur & Williams Co., Brighton, Mass.; (Classification and directory sections)
Durocon	(Integral hardener for concrete floors) Aquabar Co.; Philadelphia, Pa.
Dye-Crete	(Rubber-base paint for concrete floors) Wilbur & Williams Co., Brighton, Mass.
Ferricon	(Concrete floor hardener) Truscon Laboratories, Detroit, Mich.
Ferrocon	(Waterproofing agent) Aquabar Co., Philadelphia, Pa.
Firecrete	(Castable refractory) Johns-Manville Sales Corp., New York, N. Y.
Garcrete	(Plasticizer) Garco Products, Inc., Butler, Pa.
Hearth-Crete	(Castable refractory) Quigley Co., New York, N. Y.
Hil-Crete	(Rubberized floor enamel) Hillyard Chemical Co., St. Joseph, Mo.
Insulcrete	(Castable refractory) Quigley Co., New York, N. Y.
Kleencrete	(Masonry surface cleaner) Evercrete Corp., New York, N. Y.
Konkerit	(Masonry paint) Toch Brothers, Inc., Staten Island, N. Y.
Konset	(Admixture) Sullivan Co., Memphis, Tenn.
Kontite	(Masonry paint) Sullivan Co., Memphis, Tenn.
Korkrete	(Primer to be used prior to applying corkboard insulation) Lewis Asphalt Engineering Corp., New York, N. Y.
Kretedye	(Floor hardener and coloring agent) Aquabar Co., Philadelphia, Pa.
Mastiecrete	(Patching/resurfacing material) Glidden Co., Cleveland, Ohio
Nu-Krete	(Floor surfacing) Warren Refining & Chemical Co., Cleveland, Ohio

Plastikon	(Adhesive) B. F. Goodrich Co., Cleveland, Ohio
Refract-O-Crete	(Refractory brick) Acme Brick Co., Fort Worth, Texas
Sealcrete	(Coating) Service Industries, Philadelphia, Pa.
Selbacrete	(Surfacing and repair material) Selby Battersby & Co., Philadelphia, Pa.
Synkrete	(Cleaner for concrete floors) Synthex Products Co., New York, N. Y.
Tempacrete	(Concrete floor hardener) Fort Pitt Chemical Co., Pittsburgh, Pa.
Tremcrete	(Silicate aggregate for use with mastic flooring materials) Tremco Mfg. Co., Cleveland, Ohio
Zilicon	(Integral waterproofer for mass concrete) Truscon Laboratories, Detroit, Mich.

I also found some new information on words previously listed.

Word	New data
Aerocrete	Also (Admixture; lightweight concrete made therewith) Aerocrete Corp. of America, Rochelle Park, N. J.
Crescrete	Also (Hardener-accelerator-densifier admixture) Cressit Waterproofing Corp., Chicago, Ill.
Demicon	Also (Concrete curing and hardening) McMillan Products Co., Hazel Park, Mich.; *Engineering News-Record*, Oct. 7, 1954, p. 380
Duracrete	Also (Pozzolanic plasticizer) Aquabar Co., Philadelphia, Pa.
Nailcrete	Add (Fireproof compound mixed with cement and water; uses: nailing base, plaster, roof coating, and floor slabs) Nailable Concrete Co., New York, N.Y.

ANOTHER READER

* * * * *

I have been interested by your previous progress reports on concrete's etymological offspring. My collection of this strange phenomenon now consists of a total of 270 words. Listed below are those which have not as yet been listed in previous progress reports.

Most of these trade names have been accumulated for many years and I do not know if the firms listed still manufacture the product. Where known, I have indicated date when word was first used.

Word	Where used
Aircreter	(Nonagitating ready-mix truck) Supermix, Inc., Adrian, Mich.
Arid-Crete	Arid-Crete Corp. of Cincinnati, Cincinnati, Ohio
Art-Crete	(Concrete masonry) Art Craft Blocks, Inc.,Canton, Ohio. First use 1948
Baffle Crete	(Castable refractory)Mexico Refractories Co., Mexico, Mo. First use 1936
Basicrete	(Hydraulic-setting, chrome base, castable cement) General Refractories Co., Philadelphia, Pa. First use 1949
Camperete	(Self-setting surfacing and resurfacing materials — namely, cement and asphaltic compositions) The Camp Co., Chicago, Ill. First use 1948
Can-Crete	(Canned colored cement) The Sullivan Co., Memphis, Tenn.
Cast-Crete	(Precast concrete products) Cast-Crete Products Co., San Angelo, Texas. First use 1933
Channel Crete	(Prefabricated roof slabs) The Formigli Corp., Berlin, N. J. First use 1937
Creto	Creto Co. of California, Los Angeles, Calif.
Dense-Crete	(Accelerator) Rock-Tred Corp., Chicago, Ill. First use 1939
Durocrete	(Ready-mixed heavy duty floor repair cement) American Fluresit Co., Cincinnati, Ohio

Evercrete	(Treatment for dust-proofing and preventing lime action on oil base paints) Evercrete, Inc., New York, N. Y.
Faberete	(Prefabricated floor, wall, or roof units) Faberete Corp., Richmond, Va. First use 1947
Factocrete	Broadway Trust Co., Westminster, England
Flex-Or-Crete	Flex-Or-Crete Corp., Boston, Mass.
Furnas Crete	(Castable refractory) Mexico Refractories Co., Mexico, Mo. First use 1932
Glazcrete	(Glazed facing material of cement and aggregate) Horace A. Reeves, Ardmore, Pa. First use 1948
Guncrete	(Pneumatically applied concrete) Cement Gun Co., Allentown, Pa. First use 1914
Handicrete	(Concrete gravel mix) The Carney Co., Inc., Mankato, Minn. First use 1950
Heat-Crete	(Admixture) Products Planning Co., Pittsburgh, Pa.
Hofferete	(Waterproofing adhesive masonry mortar) George W. Hoffmann Corp., New York, N. Y. First use 1947
Kaycrete	(Expanded perlite aggregate for admixture with refactory cement) Great Lakes Carbon Corp., New York, N. Y. First use 1951
Kon Creto	Murphy Varnish Co., Ltd., Montreal, Canada
Koncrex	A. C. Horn Co., Long Island City, N. Y.
Konkrete Kore	(Concrete drill) Tilden Tool Mfg. Co., Pasadena, Calif.
Konkure	(Curing compound) J. R. Anderson Co., Los Angeles, Calif.
Latex-O-Crete	(Floor resurfacing compound) United Laboratories, Inc., Cleveland, Ohio
Lubricon	B. A. Davenport, Inc., Washington, D. C.
Masticon	Frank White, Minneapolis, Minn.
Novocrete	The Novocrete Co., New York, N. Y.
Omacrete	(Premixed concrete) Concrete Products and Material Co., Inc., Omaha, Neb. First use 1947
Presterete	(Prefabricated concrete units) Illinois Presterete Co., Plano, Ill.
Presscrete	(Grouting equipment) The Presscrete Co., Inc., New York, N. Y.
Pumacrete	(Cementitious building materials) The Ketell Co., Los Angeles, Calif. First use 1946
Red E Crete	(Premixed concrete) Kuring and Erickson, Assoc., Glendale, Calif. First use 1946
Rockercrete	(Block machine) Roy Darden Industries, Inc., Atlanta, Ga.
Rocrete	(Material used in precast building units) The Rohan Co., Inc., Garden City, N. Y. First use 1933
Sackerete	Sackerete, Inc., Cincinnati, Ohio
Steel-Crete	(Concrete and plastic compositions) Joseph Downes, Sioux City, Iowa. First use 1921
Steelcrete	(Expanded metal and reinforcing mesh) Wheeling, W. Va. First use 1907
Thermo-Con	Higgins Industries, New Orleans, La.
Trip-L-Crete	(Patching cement) Trip-L-Seal, Inc., Washington, D. C. First use 1947
Tuferete	Tuferete Co., Des Moines, Iowa. First use 1940
Unicon	(Vermiculite concrete) Universal Insulation Co., Chicago, Ill.
Vitocrete	The Portland Cement Selling and Distributing Co., Ltd., London, England

ANOTHER READER

An Application of Radioactive Tracer Techniques (LR 51-19)

Evaluation of the Relative Penetration of Fluosilicate Solution into Concrete With and Without the Presence of Wetting Agents

That the reaction between fluosilicates and calcium hydroxide (and/or carbonate) yields insoluble decomposition products has been known for many years. The utilization of this reaction for the purpose of chemical hardening of concrete surfaces by application of aqueous solutions of the fluosilicates to these surfaces has been equally well known, having originated in Germany about 60 years ago. Since that time many millions of square feet of concrete surfaces and particularly flooring have been hardened by that method all over the world.

The chemical reactions which take place, and it is important to note that they take place *in situ*, are the following

(1) $MSiF_6 + 2Ca(OH)_2 \rightarrow MF_2 + 2CaF_2 + SiO_2 + 2H_2O$

(2) $MSiF_6 + 2CaCO_3 \rightarrow MF_2 + 2CaF_2 + SiO_2 + 2CO_2$

Where M is the metal cation attached to the SiF_6 group.

It is obvious that the insoluble flourides and silica left as reaction products within the pores of the surface of a portland cement concrete slab must raise its abrasion resistance. Indeed, when the concentrations of the solutions of fluosilicates and the number of sequential applications are judiciously controlled to conform with the degree of porosity of the concrete surface to be hardened, the end result is hardly distinguishable from a vitrified surface which is virtually immune to the abrasive action of the heaviest industrial traffic. This result of course assumes that the original floor has been laid in accordance with the best principles of design.

An additional advantage accruing from the treatment is the enhanced resistance of the treated surface to acid and acid salt attack. This makes it possible for example to flush away acid spillage (on, say, plating room floors) before any erosion of the concrete can take place.

This letter relates to the evaluation of the treatment in terms of the depth to which it is effective. Many efforts have been made in the past to arrive at such an evaluation by way of suitable abrasion tests. The consensus of observation, however, in such tests has been that they are difficult to control especially in view of the fact that the hardening action of fluosilicates takes place virtually on the surface and that if any reaction below the surface is realized it is at best to a depth of only 1 to 2 mm.

The reason for this observation derives from the fact that the reaction between a fluosilicate solution and a concrete surface is instantaneous, *i.e.*, it takes place the moment that the solution contacts the surface. Surface pores are immediately filled with the metal fluorides and silica, and migration or penetration of the solution below the surface is thereby impeded and on all but the most porous surfaces (which of course are not characteristic of a good concrete floor) are substantially completely stopped.

Concordant and consistent results of abrasion tests have been difficult to obtain for other reasons such as irregularities in morphological structure, variations in size and hardness of randomly distributed aggregate, and variations in quality of concrete. Variables of this type would normally be a bar to uniform testing of any refractory and empiric substance such as concrete.

Some years ago the writer conceived the idea of lowering the surface tension of fluosilicate solutions, *i.e.*, imparting surface activity by the incorporation of suitable wetting agents, so as to obtain positive penetration of the solution into the body of a concrete slab. Upon reduction to practice there was every indication that the idea was entirely operative and a patent, U. S. Patent 2,203,302, was issued on the improvement. Various physical methods of validating the idea such as differential staining and microscopic study of treated sections of the concrete were defeated by the variables mentioned as much as by the fact that normal concrete and fluosilicate treated concrete exhibited no readily discernible differences in essential structure. Abrasion tests of both fluosilicate treated and un-

treated slabs give some indications but of an indecisive character.

Some years ago discussion of the subject with technologists of the National Bureau of Standards (where the last mentioned abrasion tests were conducted) led to a suggestion by Bruce E. Foster that a feasible approach might be through the use of radioactive tracers. Since such tracers became readily available and isotope technology had developed feasible methods, the writer commissioned the radioactive chemical staff of Foster D. Snell, Inc., to reduce the idea to practice. A summary of the report follows, and we quote.

"We were authorized by the client to undertake studies to determine the penetration of zinc fluosilicate into concrete. This was to be done . . . with and without the addition of a suitable wetting agent in accordance with the Liberthson U. S. Patent 2,203,302.

"It has been decided to use radioactive tracer technics. For this purpose ten millicuries of zinc —65 in the form of a zinc chloride solution were obtained from the Oak Ridge National Laboratories under authorization from the Atomic Energy Commission . . . Several millicuries of carrier-free $Zn^{65}SiF_6$ (zinc fluosilicate) were then prepared . . .

"A solution was then prepared containing 20 g of ordinary $ZnSiF_6 \cdot 6H_2O$ per 100 ml of solution. Enough of the radioactive material was then added to give about 58,000 counts per min per ml of solution. All counts were made with a Geiger-Muller counter . . . For the penetration studies, the stock solution described . . . was diluted with water in the proportion of 1:2, 1:1, and 2:1 . . . In one series of experiments, squares of average mortar (1:4 Ottawa sand to standard portland cement, finished to approximate the quality of class C concrete topping) were treated successively with each diluted solution. The figures in the columns 2 to 4 (Table 1) were obtained with a zinc fluosilicate solution which contained no wetting agent. The figures in columns 5 to 7 were obtained with a solution of the same concentration of zinc fluosilicate, but which also contained a solution of an anionic wetting agent supplied by the client . . . The relative specific activity found at various depths is then found by dividing the activity in suitable units by the weight of the sample counted . . . These quantities are directly proportional to the amount of radioactive reaction products to be found at those depths."

". . . The ratios should be regarded in the

TABLE 1—PENETRATION OF ZINC FLUOSILICATE SOLUTIONS WITH AND WITHOUT WETTING AGENT INTO AVERAGE CONCRETE*

No wetting agent				With wetting agent			
1	2	3	4	5	6	7	8
Depth, min	Activity, counts per min	Weight, mg	Relative specific activity, counts per min per mg	Activity, counts per min	Weight, mg	Relative specific activity, counts per min per mg	Wetting agent / No wetting agent
0 (surface)	598 322	18.0 9.4	33.2 34.3 mean = 33.8	745 670	22.2 21.2	33.6 31.6 32.6	1.0
0 to 1	1313 1209	89.5 90.8	14.7 13.3 mean = 14.0	1182 1150	72.1 73.7	16.4 15.6 16.0	1.1
1 to 2	143 139	496.0 502.9	0.288 0.276 mean = 0.282	566 565	208.6 230.3	2.72 2.45 2.58	9.1
2 to 5	8 3	652 686	0.012 0.004 mean = 0.008	101 93	472 433	0.224 0.215 0.220	27.5
5 to 7	0 0	742 813	0 0 mean = 0	2 1	520 675	0.004 0.001 0.002	∞†

*Treatment: 1 min with 1:2, 2 min with 1:1, 3 min with 2:1.

†Calculated ratio is infinite, but counts are so low, they approach or equal probable error. However, observed data, together with extrapolation of data at lesser depths, indicates a probable ratio of the order of 100 or more.

light of the fact that the chemical reaction that takes place is instantaneous on surface contact . . . For the first millimeter or so, the ratios are exceedingly close to unity. However, the surface active solution does indicate greater penetration and hence greater deposition of reaction products even into the topmost millimeter. The picture then changes dramatically, and, at a depth of 1 to 2 mm, *the addition of the wetting agent has increased the amount of zinc fluosilicate reacting with the concrete at that level by a factor of ten.* Even more spectacular ratios are obtained at 2 to 5 mm . . .

"In brief then it may be said that the presence of a wetting agent in a zinc fluosilicate solution increases the penetration of this substance into concrete in two ways: *the solution will penetrate deeper and reaction products will be deposited in greater quantities.* . . . In the range of 1 to 2 mm, the wetting agent increases the amount of zinc by a factor of about ten in the case of average concrete. At greater depths, this effect is more marked still. *The results obtained definitely indicate the greater advantage of the wetting agent, which is almost spectacular in the case of average concrete.*

"The results obtained in these investigations are completely trustworthy. They would have emerged even more vividly had much higher specific activities been used. This was not done because of the much greater radioactive hazards involved . . ."

Acknowledgment is due technologists of Foster D. Snell, Inc., for their valuable cooperation and suggestions in the discussions which led to the execution of the problem as outlined.

Leo Liberthson, Technical Director, Building Products Division, L. Sonneborn Sons, Inc., New York, N. Y.

Current Reviews*

of Significant Contributions in Foreign and Domestic Publications

Bridges

Prestressed slab bridges (in Hungarian)

E. Bolcskey and A. Petur, *Melyepitestudomanyi Szemle*, V. 2, No. 9, Sept. 1952, pp. 456-462
Hungarian Technical Abstracts V. 5, No. 3, 1953

Use of prefabricated, prestressed units in the construction of slab bridges is discussed. Units were manufactured according to the Hoyer system. Joint action of the in situ concrete deck slab and the prefabricated units is ensured by the bond of the concrete and also by bent-up reinforcement. Structures of this type are suitable for spans of 2 to 5 m. Openings between 5 and 7 m are spanned with slabs that have edge beams. In this case the structure acts as a two-way slab supported in one direction by the edge beams. The maximum calculated value of the bond stress between the in situ concrete and the prestressed units before failure was 8.2 kg per sq cm, whereas under actual loading this value did not exceed 2 kg per sq cm. This joint action proved satisfactory in the case of dynamic loading as well. The economy in steel attributable to this type of structure reached 48 and 51 percent for 3- and 7-span bridges, respectively.

Composite bridge structures in Turkey (Neke beleske uz projekte mostova za Tursku)

M. Momirov, *Nase Gradevinarstvo* (Yugoslavia), V. 8, No. 6, 1954, pp. 159-162
Reviewed by J. J. Polivka

Description of five bridges built in Turkey. Three of them consist of three free spans of 46 ft, one is a Gerber-type with center span of 71 ft and two side spans of 53 ft, and the fifth bridge is a combination of three continuous spans of 59 ft and a cantilever three-span type, the center span measuring 130 ft and the side spans 63 ft each. Typical width of the deck is 36 ft, including two sidewalks 5 ft wide. Design and analysis are discussed with reference to previous studies and publications by Dischninger, Fröhlich, Esslinger, Sonntag, Sattler, Reinitzhuber, and Leonhardt.

Four large prestressed concrete bridges in The Netherlands (in Dutch)

J. J. B. J. Bouvy and G. F. Janssonius, *Cement* (Amsterdam), No. 17-18, pp. 278-285; No. 19-20, pp. 320-323, 1954
Reviewed by John W. T. Van Erp

Some large prestressed bridges were built in 1953 in The Netherlands, all of them demonstrating the savings possible over conventional reinforced concrete, amounting to 20 percent in one instance (see example 4). Examples 2, 3, and 4 were continuous structures.

1. *Bridge at Pannerden*—12 simple spans of 51 ft. Post-tensioned, steam-cured beams of T-section, constant height. The 96 identical beams were prefabricated at the job site. Tensile stresses in the concrete up to 210 psi during erection only, taken care of by mild steel reinforcement. Lateral prestressing of 25 ft long cables by jacking from one side of the bridge only. Quantities: concrete—630 cu yd; prestressing steel—23 tons; and mild steel—17 tons.

*A part of copyrighted Journal of the American Concrete Institute, V. 26, No. 8, Apr. 1955, *Proceedings* V. 51. Address 18263 W. McNichols Rd., Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parenthesis following the English title. Copies of articles or books reviewed are **not** available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

2. *Bridge at Nijmegen*—three spans of 85 ft. Post-tensioned girders of T-section, constant height. A new method for establishing continuity (on which patents have been applied for) was used and proved to be simple. The beams are about ⅛ longer than their spans and thus cantilever out over the piers into the next spans. They are staggered in plan so that their cantilevered ends overlap each other. These ends are pressed together by lateral prestressing and the friction developed between the beam-side faces at the supports is ample to take care of negative moments. Quantities: concrete—850 cu yd; prestressing steel—39 tons; and mild steel—19 tons.

3. *Bridge at Leimuiden*—Five continuous spans, biggest center span; 100 ft. Post-tensioned girders of T-section, varying height (curved haunches). Center span girders were fabricated in a casting yard on the river bank and prestressed for dead load only. The other cast-in-place girders were also prestressed for dead load only with all cables following draped patterns. Joints were at points of inflection. After erection, continuity was effected by continuity cables running straight from end to end (270 ft). Lateral prestressing cables in this skew bridge followed the skew, running parallel to abutments and piers. Quantities: concrete—670 cu yd; prestressing steel—28 tons; and mild steel—24 tons.

4. *Bridge over the Amstel River*—Five spans of 75, 112, 186, 112, and 94 ft, with the three large spans continuous; four lanes wide for fast traffic plus two for slow traffic (mostly bicycles) and two sidewalks; total width 89 ft. Girders post-tensioned, T-sections of varying height, prefabricated in sections in a casting yard near the job site. Joints were at the points of inflection and scaffolding for erection was needed only at these points. Steel forms were used, externally vibrated, and a great number of test cubes, 12 per girder section, were made in steel molds built integrally with the girder forms. Test specimens were thus as identical to the concrete in the girders as possible. Continuity was obtained by the use of continuity cables. During continuity prestressing the levels of the girder tops were constantly surveyed as a check on deformations and they proved to be in fairly good agreement with predicted deflections, notwithstanding uncertainties as to the modulus of elasticity of the various girder sections of different age.

Construction

Construction program of Philadelphia Suburban Water Co. meets unprecedented demand

FRANCIS S. FRIEL, *Civil Engineering*, V. 24, No. 2, Feb. 1954, pp. 41-45

Construction details are given on transmission, water storage, and distribution-system facilities. Of particular interest is description of concrete storage tanks. The tanks are 126 ft in diameter and 54 ft 9 in. high. Tank walls were prestressed vertically and horizontally. Each dome, 2½ in. thick increasing to about 10 in. at the outer edge, was shotcreted.

Nechako-Kemano-Kitimat development

The Engineering Journal (Montreal), V. 37, No. 11, Nov. 1954

Reviewed by ARON L. MIRSKY

Entire issue is devoted to papers on this power-cum-aluminum smelting development: hydraulics, Kenney dam, underground powerhouse, penstocks, transmission lines and towers, Kitimat harbor, smelter, and even the effect of the power development on fisheries.

Design of a new shipyard

Engineering (London), V. 178, No. 4618, July 30, 1954, pp. 154-156

Reviewed by ARON L. MIRSKY

Precast prestressed members were used extensively in the construction of buildings, paving slabs, and other items for a new shipyard at Lowestoft. Choice of precast concrete was based primarily on unfavorable experience with steel structures in this locality.

Problems in construction of underground water reservoirs (Nekaj konstruktivnih proproblemov pri gradnji podzemnih vodnih rezervoarjev)

FRANC ADAMIC, *Gradbeni Vestnik* (Yugoslavia), V. 6, No. 19-20, 1953, pp. 193-196

Reviewed by J. J. POLIVKA

Discusses some problems encountered in design and construction of underground reservoirs in reinforced concrete, *e.g.*, most economical shape for various capacity (circular, rectangular), effect of the bearing capacity of soil, various types of walls, roof and floors, drainage, watertight gates, properties of concrete, and surface treatment. Describes use of

prestressed concrete in a 50-ft diameter circular reservoir in Crawley, England. It is 27 ft deep and has a 2 in. thick shotcreted dome shell. Walls are 5 in. thick, and the 2-in. bottom slab rests on a 3-in. concrete base.

Derrick-hoisted forms save time and money

M. J. Colarusso, *Civil Engineering*, V. 24, No. 1, Jan. 1954, pp. 48-49

A "form-lift" method was used in erection of 175 x 75-ft four-story reinforced concrete building. The method basically consisted of hoisting external wall form panels from floor to floor as each section was completed. Form panels were standard plastic-coated ⅝-in. plywood of the stud-and-wale type, in basic section sizes of 37 ft wide and 14 ft high. Two 22 ft high, 5-ton capacity hand derricks were used to lift a panel.

Dams

Progress in the manufacture of plain concrete, with especial consideration to construction of water power dams (in Swedish)

F. Tolke, *Betong* (Stockholm), V. 39, No. 1, 1954, pp. 1-33

Author's Summary

Because of the increasing demands on the strength of concrete and especially by those caused by the development of water power in the Alps, manufacture of concrete in Europe has made notable progress during the last few years. This is described by regarding some remarkable examples. Progress is shown in the field of weatherproofness and durability of concrete. Also discussed are: behavior of internal stresses in concrete structures, porosity of cement mortar, importance of cement class and coarseness of material, influence of the finest particles and of air entrainment on the behavior of concrete, and the development of cracks by secondary chemical processes.

Design

Girder on elastic foundations (in Croatian)

Joze Staric, *Nase Gradevinarstvo* (Yugoslavia), V. 8, No. 7, 1954, pp. 201-208

Reviewed by J. J. Polivka

Referring to previous research by Kammüller and Kögler-Scheidig, the author develops equations of influence lines of settlements, bending moments, and shears for two cases: (a) continuous foundation slab; (b) foundation slab with a hinged joint in the middle.

Design of cylindrical shell roofs

J. E. Gibson and D. W. Cooper, D. Van Nostrand and Co., Inc., New York, N. Y., 1955, 185 pp., $8.50

Essentially a mathematical treatment of cylindrical shell roofs. The book is suitable for advanced structural engineering courses at the graduate level. Derivation of theory is similar but more difficult to follow than that given in the appendix of *ASCE Manual of Engineering Practice No. 31*. For practicing designers, this book falls far short of the manual in usefulness. Material included which is not available in the manual includes a chapter on the design of prestressed edge beams for cylindrical shell roofs, north-light shell roof design, and photoelastic studies on end diaphragms in cylindrical shells. The mathematical approach which is strongly emphasized throughout the book, somewhat to the detriment of practical design considerations, works up to a chapter on solution by matrices which the authors state is suitably arranged for programming for an automatic electronic calculating machine.

Design of gasholder tanks

Indian Concrete Journal (Bombay), V. 28, No. 9, Sept. 1954, pp. 368-369, 397

Design for a reinforced concrete gasholder tank, suitable for a small gas producing plant, is presented.

Theory of plasticity in reinforced concrete (Teorija plasticiteta u armiranog betona)

H. Chaemer, Izadev. Predur. Minist. Gardjev., Belgrade, Yugoslavia, 1951, 309 pp.

Applied Mechanics Reviews June 1954 (Brandes)

In many countries the methods for determining dimensions of cross sections for reinforced concrete members are changing from those of the theory of elasticity (*n* method of admissible tensions) to methods resting on the plastic behavior of steel and concrete in method free from *n* rupture.

Author gives a clear and comprehensive summary of the fundamental laws proving the validity of this form of calculation. The load capacity on the various types of loading and the degree of safety are carefully discussed on the basis of results given in earlier chapters. At the end of the book, auxiliary tables for the determination of cross section dimensions are given.

A chapter treats the load capacity of statically indeterminate systems (beams, girder grillages, frames, and arches); another

deals with crosswise reinforced slabs and flat slabs. Methods are developed which enable more rapid calculation than is possible with methods based on the theory of elasticity. A bibliography contains about 100 references.

Energy methods of statics applied to linear boundary value problems (in English)

S. T. A. Odman, *Betong* (Stockholm), V. 39, No. 1, 1954, pp. 35-51

Author's Summary

Some mathematical concepts used in solving boundary value problems are described in the first chapter. The statical interpretation of these concepts is illustrated by examples. The second chapter demonstrates the connections between methods based on the principles of minimum of error potential and minimum of potential energy, on the principles of virtual forces and virtual displacements, and on the theorem of Castigliano and the law of conservation of energy. Mention is also made of the requirements for obtaining identical results by means of these methods.

Lateral buckling of tied arches

William George Godden, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 3, No. 2, Aug. 1954, pp. 496-514

Reviewed by Aron L. Mirsky

In the "Stabbogen" type of arch, the arch rib is relatively slender (about a horizontal axis) compared to the stiffening girder (or bridge deck) and hence may be treated as a pure compression member. Paper presents a theoretical analysis based on Rayleigh's principle, and describes experimental investigations on steel models. Results indicate that hanger pulls exert a considerable stabilizing influence on arch ribs, which are thus considerably more stable laterally than chords of trusses.

Graphical determination and arrangement of shear reinforcement and stirrups in reinforced concrete beams (Graficno dimenzioniranje posevne armature in stremen armiranobetonskih nosilcev)

Srdjan Turk, *Gradbeni Vestnik* (Yugoslavia), V. 6, No. 23-24, 1953, pp. 54-64

Reviewed by J. J. Polivka

Two graphical methods are presented (1) based upon the lines of maximum bending moment, whereby the shear values are determined by the angle of tangents to the moment lines at any particular cross section (maximum positive and negative bending moments), and (2) based upon the influence lines of bending moment and shear. Classical solutions are presented in a graphical way for beams with constant and variable cross sections. A special chapter is devoted to graphical design of stirrups, their dimensions and spacing.

Design of grass bins (elevators) in reinforced concrete (O projektiranju in gradnji silosov za kislo krmo)

Martin Orban, *Gradbeni Vestnik* (Yugoslavia), V. 6, No. 23-24, 1953, pp. 70-74

Reviewed by J. J. Polivka

Discusses the work of A. P. Pemington at the New Jersey Agriculture Experiment Station of the U. S. Department of Agriculture on design of silos for grass. Important factors discussed are the relation between capacity and dimensions of the elevator, most economical dimensions for a given capacity, foundations, wind pressure on high elevators, construction methods, and maintenance.

Study of the bowstring arch having extensible suspension rods and different ratios of tie-beam to arch-rib stiffness

Sirilakn Chandrangsu and Stanley R. Sparkes, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 3, No. 2, Aug. 1954, pp. 515-563

Reviewed by Aron L. Mirsky

Three methods of analysis are presented and discussed: (1) exact, including the effect of the rigidity of the arch; (2) approximate, assuming pin connections at ends of tie rod, inextensible suspenders, and relatively flexible arch ribs; and (3) membrane analogy, with the suspenders replaced by a continuous elastic membrane. Influence coefficients based on strain-energy relationships are used for the first two, using "increment transformations" to simplify the calculations; Fourier series are used in the third. Results were checked experimentally using a loaded celluloid model.

Anomalies and fallacies in the current elastic theory of flexure as revealed in its application to reinforced concrete structures. I—New doctrine of elasticity

T. Muthiah, Madras, India, 1954, 19 pp.

Reviewed by Aron L. Mirsky

A lecture delivered at the Institution of Engineers (India), Madras Centre, Feb. 27, 1954. Author's proofs of validity of elastic principles for reinforced concrete design are based on theological grounds; his new doctrine of elasticity is based on Hindu philosophy. Two further lectures, on a new concept of force and a new elastic theory of flexure, are promised.

Analysis of multiple frame structures with variable cross sections by the method of substituted members (Rjesavanje okvira sa stapovima promjenjivih preseka pomocu zamjenjujucih stapova)

V. Andrejev and D. Moric, *Nase Gradevinarstvo* (Yugoslavia), V. 8, No. 7, 1954, pp. 177-181
Reviewed by J. J. Polivka

The elastic properties of individual members rigidly connected are expressed by fixed points, as developed by Ritter, Suter, and other authors. The method is demonstrated by numerical examples.

Analysis of multiple-story frames subjected to horizontal loading (in Croatian)

Zivojin Hiba, *Nase Gradevinarstvo* (Yugoslavia), V. 8, No. 7, 1954, pp. 195-201
Reviewed by J. J. Polivka

Three methods are thoroughly discussed and compared on the basis of a numerical example: (1) of Charles E. Reynolds (*Reinforced Concrete Designers Handbook*, London, 1948); (2) of J. Eisenmann (*Der Bauingenieur*, V. 28, No. 6, 1953); and (3) of Ralph W. Stewart ("Traverse Method for Analyzing Continuous Structures," 1948). Author expresses the opinion that the treated problem is not finally solved, and makes suggestions for simplifications of the analysis, especially time-saving for frame structures with relatively slender columns.

Shell structures

A. L. Delaney, *Journal*, Boston Society of Civil Engineers, V. 41, No. 4, Oct. 1954, pp. 405-417
Reviewed by Aron L. Mirsky

Simplified exposition of how shells function, emphasizing the geometrical and physical relationships involved. A useful introduction to the hitherto uninitiated to *ASCE Manual of Engineering Practice No. 31*, "Design of Cylindrical Shell Roofs."

Design of tee-beams

H. J. Hopkins, *Concrete and Constructional Engineering* (London), V. 49, No. 10, Oct. 1954, pp. 303-306
Author's Summary

Most methods of designing T-beams require the depth to be determined by trial and error with the aid of charts, or the use of complicated formulas. There is a risk that these procedures may cause the designer to lose sight of the physical aspects of design. The simple method of obtaining the minimum depth described here emphasizes the physical consequences of each step. It can be extended to the design of all T-beams, for if the depth is greater than the minimum required it is necessary to determine only the lever arm and the area of steel.

Materials

Refractory concrete for refinery vessel linings

J. F. Wygant and W. L. Bulkley, *American Ceramic Society Bulletin*, Aug. 1954, pp. 233-239
Authors' Summary

Refractory concretes are usually composed of aluminous hydraulic cement and calcined clay aggregates. Those which are intended especially for use at temperatures below 1500 F are useful as lining materials in refinery vessels and transfer lines, to resist erosion, abrasion, and certain types of chemical attack, and (with lightweight aggregates) to reduce metal shell temperatures. The best concretes for these refinery uses develop high strength within three days, are fairly volume stable and highly resistant to thermal shock, and maintain relatively high strengths at service temperatures. These and other properties, such as erosion and abrasion resistance, depend upon the type and particle size gradation of the aggregate, the proportions of cement and aggregate, and the installation methods. Proper installation is extremely important. Attention must be given to regulation of water content, good mechanical procedure in application (particularly in pneumatic application), prevention of segregation, and the best possible curing conditions. The concrete designer must consider limitations on section thickness, and must provide suitable reinforcement. Basic data for the design of reinforcement for concrete to be used at elevated temperature are scanty. Design criteria for structural concrete cannot be extrapolated safely for high temperature applications. Also, application methods for refractory concretes usually differ from those used with structural concrete.

Effect of grading and shape on bulk density of concrete aggregates

D. S. Moscrieff, *Magazine of Concrete Research* (London), No. 14, Dec. 1953, pp. 67-70
Author's Summary

Laboratory tests have shown that variation of grading within the limits permitted by the British Standard 882:1944 for concrete aggregates results in differences of as much as 12 lb per cu ft in the compacted bulk density of coarse aggregates, and 13 lb per

cu ft in Class A and 31 lb per cu ft in Class B fine aggregates. It was also found that the bulk densities of different gradings of a rounded aggregate were between 5 and 10 percent higher than those of the corresponding gradings of an angular aggregate with the same specific gravity.

It is concluded that errors in the proportioning of concrete designed by weight and batched by volume might arise if a constant value of bulk density were assumed for an aggregate complying with the British Standard, and that it is necessary in practice to measure the bulk density rather than to estimate it from available data.

Is blast furnace cement subject to false set? How to prevent it. (Le ciment de haut fourneau est il sujet a fausse prise? Comment la prevenir)

LEON BLONDIAU and IVAN BLONDIAU, *Revue des Materiaux de Construction* (Paris), No. 466-467, July-Aug. 1954, pp. 215-222

Reviewed by PHILLIP L. MELVILLE

Report on a series of tests on false set of blast furnace slag cement. Difficulties were encountered in evaluating false set. Its relation to the condition of the gypsum appears critical because calcium sulfate is dehydrated to a certain extent during grinding. Blast furnace slag cements have unstable setting when containing gypsum in the semi-hydrated state.

Deleterious constituents of Indiana gravels

D. W. LEWIS and EDUARDS VENTERS, (discussion by F. E. LEGG, JR.) *Bulletin* No. 94, Highway Research Board, $0.30

Reports results of tests on Indiana gravels ranging from good to poor in field performance in portland cement concrete. Gravels were separated by liquid flotation into various specific gravity ranges. Gravel fractions thus obtained were tested to determine absorption, specific gravity, degree of saturation, lithologic composition, and durability in air-entrained concrete subjected to freezing and thawing.

Results show that principal deleterious constituents of Indiana gravels are sandstones and cherts. They are characterized by low specific gravity, high absorption and degree of saturation, and produce nondurable concrete when used as the coarse aggregate. Concrete durability can be improved by heavy-media separation of the aggregate. Greater durability is obtained as the specific gravity at which separation is made is increased.

Bulletin also contains a discussion by F. E. Legg, Jr., assistant supervisor, Michigan State Highway Department Testing Laboratory. Discussion describes results to be obtained by the heavy-media separation method applied to gravels from principal sources in Michigan to enable them to meet Michigan State Highway Department specifications.

An x-ray examination of the structure of vermiculites

A. GRUDEMO, *Handlingar* (Proceedings) No. 22, Swedish Cement and Concrete Research Institute at the Royal Institute of Technology, Stockholm, 1954, 56 pp.

AUTHOR'S SUMMARY

A short account is given of the relationships between the structure of the hydrous magnesium silicate vermiculite and the structures of other silicate hydrates such as certain cement hydration compounds. A short review of previous data on the structure of vermiculite is given. Three different specimens of vermiculite minerals are selected for study. Various properties and quantitative data of these minerals are presented, such as chemical composition, cation exchange capacities, optical properties in combination with Laue symmetry, and values of density and linear x-ray absorption coefficient. The unit cells are determined and lattice symmetry relationships are discussed. The techniques applied in x-ray intensity measurements are described and analytical expressions of absorption correction factors necessary in structure factor calculations are given. The atomic distribution in different crystals is studied by means of superposition Patterson and Fourier syntheses, the most probable structures being sought by minimizing the discrepancies between observed and calculated structure factors.

A warning regarding air entraining agents (En varning vid tillsattning av luftillforande medel)

P. NYCANDER, *Betong* (Stockholm), V. 38, No. 4, 1953, pp. 247-250

AUTHOR'S SUMMARY

In the manufacture of air-entrained concrete it appears that the reduction in strength which may be caused by an increase in air content is not always taken into consideration. The relation between compressive

strength and air content is discussed and illustrated by experimental data. Emphasis is laid upon the importance of continual control of the air content.

Field substitution of fly ash for a portion of cement in air-entrained concrete

GUY H. LARSON, *Proceedings*, Highway Research Board, V. 33, 1954, pp. 258-264

AUTHOR'S SUMMARY

A field investigation was made of the effects of substituting fly ash for a portion of the cement in air-entrained pavement concrete. Requirements for the concrete were standard in all respects except that they called for using 20 lb of fly ash with each 75 lb of cement and air content of 2.5 to 6 percent.

Test sections were placed with the standard paving mixture using a nominal 5.4 sacks of air-entraining cement per cubic yard, the mixture with the fly ash substitution, and the latter mixture with air-entraining agent added to restore the air content to nearly that of the standard. Tests included strength and durability tests on field and laboratory mixed concrete, and strength tests on cores taken from the pavement when it was 39 months old.

The results showed that substituting fly ash for about 20 percent of the cement increased the strength of the concrete at ages beyond 28 days but reduced the air content and the resistance to freezing and thawing. Restoring the air content of the fly ash concrete reduced the strength but restored the resistance to freezing and thawing to a large extent.

Strength tests on the cores showed that in the pavement both the concrete with the fly ash substitution and reduced air content and the fly ash concrete with air content restored were substantially stronger than the standard concrete.

Pressure prehydration increases strength of hydraulic cement mortar specimens

THOMAS H. WHITFIELD, JR., *Civil Engineering*, V. 23, No. 9, Sept. 1953, pp. 57-58

Strength of hydraulic cement products depends to a great extent on the degree of hydration of the cement. The more intimate the combination of water and cement, the greater the strength of the resulting product. The author conceived the idea of obtaining a more intimate combination by employing pressure to force water into the particles of cement. Three limited series of tests were made to test the validity of the idea. Average strength of the treated specimens was found to be greater than that of untreated specimens.

Pavements

Structural design of concrete roads

F. N. SPARKES, *Magazine of Concrete Research* (London), V. 6, No. 17, Sept. 1954, pp. 61-70

Reviews present knowledge of the thickness, length, and width of concrete pavement slabs, use of reinforcement, and the functions and design of joints.

Testing for construction and evaluation of airport pavements

CLIFFORD J. CHUNN and FRANK M. BELL, *Proceedings*, ASCE, Separate No. 418, V. 80, Mar. 1954, 22 pp., $0.50

Describes investigational methods and procedures employed by the U. S. Army Corps of Engineers in connection with the design, construction, and evaluation of airport pavements.

Edwards Air Base taxiways

Pacific Road Builder and Engineering Review, Apr. 1954, p. 13

ARBA TECHNICAL INFORMATION DIGEST
May 1954

Concrete construction at Edwards Air Force Base consists of a 300 x 15,000-ft runway, an east connecting taxiway and warm-up apron, and about 40 acres of parking apron. The concrete runway will be 17 in. thick except at each end where the thickness will be increased to 19 in. to accept the heavier loads caused by standing and slow moving craft. The runway is being placed in 25-ft lanes. There are no expansion joints, and contraction joints are cut at 25-ft intervals to a 3 in. depth with a 16-blade diamond set concrete saw. The joints are then filled with a jet fuel resistant mastic compound.

Precast concrete

New systems of prefabrication of industrial hall structures

A. MAJOR, *Acta Technica Academiae Scientiarum Hungaricae* (Budapest), V. 8, No. 1-2, pp. 1-23

AUTHOR'S SUMMARY

Main principles of the Major system of prefabrication were published in *Industrial Hall Systems, Design and Construction*, 1950.

Further details appeared in *Steel and Reinforced Concrete Construction*, 1952, and in "Prefabrication in the Hungarian Practice," *Publications of the Hungarian Academy of Sciences*, V. 6, No. 1-2, 1952. The present paper, summarizing new developments, deals with a partial prefabrication system combined with traveling scaffolding and with total prefabrication. According to this system, frames cast vertically with regard to their final position are lifted on transport towers by winches and carried to their scheduled place. The requirements for forming and scaffolding material are small and, in winter, the concreting site can be easily roofed. Prefabrication and construction of foundations for building and machines can be carried out at the same time. No special concreted site for prefabrication is needed. Prefabrication of industrial structures is possible in half the time and 10-12 percent of the total cost can be saved. The system, developed also for post-tensioned concrete structures, avoids the use of expensive lifting machinery and shows in principle a new stage of development in prefabrication compared with conventional systems.

Prestressed concrete

Precast and prestressed members form skeleton of multistory office building

FRITZ KRAMRISCH, *Civil Engineering*, V. 24, No. 1, Jan. 1954, pp. 54-55

Construction of three-story office building was facilitated by use of precast and prestressed concrete columns and beams. Building is 60 x 365 ft.

Principles and practical applications of partially prestressed concrete

P. W. ABELES, *The Reinforced Concrete Review* (London), V. 3, No. 5, 1954, pp. 249-301 (including discussion)

Reviewed by WILLIAM R. LORMAN

Presents essential features of fully-prestressed and partially-prestressed concrete; explains more fully bridge, roof, mast, composite slab, road, and indeterminate structures comprised of partially-prestressed concrete; and includes an appendix of basic design formulas supplemented by two design-computation examples of 40- and 80-ft span roof beams. Main theme is to obtain more resilience in the structure by increasing the allowance for permissible cracking. Although partial prestressing permits visible crack formation such cracking does not develop necessarily under working loads. Cracks developed by temporary high loading are harmless provided they close under normal loading and provided repetitive high loads are not extreme. Application of this design leads to reduced prestressing forces and results further in certain technical advantages when post-tensioning; the high-strength steel used is the same type as that employed normally with fully-prestressed systems but greater tensile stresses in the concrete occur under working loads. Bibliography consists of 21 references.

Discussion, by nine authorities, includes definition of the term "partial prestressing," tendency of debris-filled pavement cracks to contribute to higher compressive stresses, economics of partial prestressing, concrete tensile stresses as high as 900 psi, waterfront-structure fenders that incorporate this type of prestressing and are subjected to sea water, and fatigue tests of concrete slabs fabricated with high-sulfate-resistant metallurgical cement.

Friction and jack losses in prestressed concrete (Friktions- och domkraftsforluster vid forspand betong)

A. BERGFELT, *Betong* (Stockholm), V. 39, No. 1, 1954, pp. 53-65

Reviewed by EIVIND HOGNESTAD

Results of tests made at Gothenburg are reported concerning a simple-span girder bridge prestressed by the Freyssinet system. The friction loss at cable bends may be obtained by determining the force transmitted from one end of a cable to the other, or by measuring stress and elongation at one end only. Test results indicate a friction coefficient of about 0.5. For straight cables the loss was about 35 lb force per linear ft. Loss in the hydraulic jacks was about 10 percent.

Development of prestressed hollow pile

British Constructional Engineer (London), V. 4, No. 4, Mar. 1954, pp. 43, 46

A newly developed prestressed concrete hollow pile has a 12-in. diameter cavity throughout its length. Constructed of 5 ft long, 16 x 16-in. sections, the piles are 50 ft long. (See also "Current Reviews," ACI JOURNAL, *Proc.* V. 51, Dec. 1954, p. 392; Feb. 1955, p. 585).

Prestressed concrete construction methods

J. M. ANTILL, *Commonwealth Engineer* (Melbourne), V. 41, No. 5-6; Dec. 1953, pp. 181-186; Jan. 1954, pp. 221-225

A basic discussion of prestressed concrete construction, in two parts. Part 1 deals

with production of high strength concrete, classifying them into five strength groups; steam curing; and types of prestressed construction. These types are (1) precast complete, (2) precast in sections, (3) cast in place, and (4) composite.

Part 2 discusses prestressing systems, formwork, prestressing techniques, and follows up with erection examples of actual construction projects.

Composite partially-prestressed concrete slabs. Strength under repeated and static loading

P. W. ABELES, *Engineering* (London), V. 178, No. 4628, Oct. 8, 1954, pp. 464-468

Reviewed by ARON L. MIRSKY

After outlining differences between brittle and ductile materials, and reasons for preferring ductile yet resilient behavior, author reports results of static and fatigue tests in detail. His nine conclusions may be briefly summed up in the statement that the slabs acted as expected.

Prestressed concrete

W. J. JURKOVICH, *Proceedings*, Sixth California Street and Highway Conference, The Institute of Transportation and Traffic Engineering, University of California, 1954, pp. 99-103

A general discussion of the increasing use of prestressed concrete in highway work. Several advantages are cited and some typical cross sections are illustrated.

New double-loop anchorage of endlessly placed prestressing wires (Neue Doppelschlaufverankerung von endlos verlegten Spanndraehten)

RUDOLF HEMMLER and REINHARD BAUER, *Die Bauzeitung* (Stuttgart), V. 59, No. 8, Aug. 1954, pp. 280, 302

Reviewed by ARON L. MIRSKY

Brief description of new method of anchoring wires by shaping them into two loops or curls which are embedded in concrete. Prestressing is applied by jacks inserted between this "head" and the beam proper. Advantage claimed is ability to absorb the ever-increasing prestressing forces consequent upon increasing spans and loads and decreasing number of girders per bridge.

Design and construction of a prestressed concrete framed transit shed for the Port of London Authority

NAHUM N. B. ORDMAN and IVAN S. S. GREEVES, *Proceedings*, Institution of Civil Engineers (London), Part III, V. 3, No. 2, Aug. 1954, pp. 409-448 (including discussion)

Reviewed by ARON L. MIRSKY

Various factors, especially the saving in steel, led to selection of prestressed concrete portal-type frames on conventional reinforced concrete floor, with aluminum alloy roofing and siding, for this twin shed, which is 257 x 75 ft in plan in one bay and 185 x 75 ft in the other.

A brief description of this shed was previously given by George A. Wilson ("Port of London Authority Engineering Works, 1952," *Proceedings*, Institution of Civil Engineers, Oct. 1953, pp. 551-604; "Current Reviews," ACI JOURNAL, May 1954, *Proc.* V. 50, p. 813).

Preliminary investigation of the use of fiberglass for prestressed concrete

IVAN A. and ANDREW RUBINSKY, *Magazine of Concrete Research* (London), V. 6, No. 17, Sept. 1954, pp. 71-78

AUTHORS' SUMMARY

Discusses the feasibility of using fiberglass reinforcement for prestressed concrete, summarizing the physical properties of fiberglass and some experimental work. It also gives practical suggestions for future research and concludes with a comparison between fiber-glass and conventional steel reinforcement.

One example of prestressing—construction of floors in a large Paris department store (in French)

STEPHANE CLAUDE and P. LAMBERT, *Supplement to the Annales de l'Institut Technique du Batiment et des Travaux Publics* (Paris), No. 75-76, Mar.-Apr. 1954, Series: Prestressed concrete (No. 18), 16 pp.

Two halls serving as light shafts in a large department store were converted to usable floor space—six floors at the level of existing floors. The columns, girders, and floor slabs were prefabricated in elements whose weight did not exceed 3527 lb. They were then assembled in place inside the store by prestressing methods. The floor units supported by the girders were double-ribbed slabs. Only the basement columns and foundations were erected in place by traditional methods

Properties of concrete

Heat of hydration in concrete

ERIK RASTRUP, *Magazine of Concrete Research* (London), V. 6, No. 17, Sept. 1954, pp. 79-92

AUTHOR'S SUMMARY

Traces the development of a function between time and temperature, which makes it possible to refer the process of hardening of concrete at varying temperature to hardening at constant temperature. It is further shown that it is possible to predetermine the temperature of concrete during hardening

when properties of concrete, sizes and shape of the specimen, insulation, and external temperature are known. The time temperature function can also be applied to the development of strength. The validity of the theory is demonstrated by the results of tests upon 27 American and three Danish cements.

Tubular growths on the surface of concrete. A study of porosity in deposited granular beds

J. Kolaszewski, *Engineering* (London), V. 178, No. 4631, Oct. 29, 1954, pp. 558-559

Reviewed by Aron L. Mirsky

Tubular growths were found on fresh concrete surfaces of pile-cap foundation at Marchwood power station in May 1953. Phenomenon proved to be similar to that previously studied by author in case of sands in suspension: when a loose wet material settles to such an extent that a few inches of water can be found standing on the surface, the water in the mix, in moving upward, not only forms vertical channels which remain in the set concrete, but carries with it particles of the settling suspension which form volcano-like craters which are later built up to the surface of the free-standing water by products of the chemical reactions in the concrete. No recommendation as to a possible cure is made by author, but one should be obvious.

Permeability tests on concrete using prismatic specimens (Provning av vattentathet hos betong med prismaformade provkroppar)

P. Nycander, *Betong* (Stockholm), V. 38, No. 4, 1953, pp. 251-260

Author's Summary

Swedish concrete standards for concrete permeability tests specify specimens consisting of solid cylinders or cylinders with brick cores. Both methods have their shortcomings and neither is convenient. The Government Testing Institute has examined other methods for determining the permeability of concrete and has resolved upon specimens consisting of solid prisms, measuring 25 x 25 x 12.5 cm, which are subjected to a water pressure of 8 kg per sq cm. The water pressure is applied to the face of the specimen which was at the bottom during casting. The area upon which the pressure acts is limited by a rubber sealing ring with a 17-cm internal diameter. The concrete is considered impermeable if no part of the upper face becomes moist after a test period of 24 hr.

Experimental results show that prismatic specimens give considerably less dispersion in permeability for a given type of concrete than do other types of specimens. The suitability of the prism as a specimen for compressive strength testing has also been investigated. Cubes 20 cm on a side of the same concrete showed 16 percent higher strength than the prisms. The dispersion in strength was about the same for cubes and prisms.

Permeability tests on prisms have proved more suitable, in various respects, than tests on standardized specimens.

General

National fire codes. V. 3—Building construction and equipment

National Fire Protection Assn., Boston, Mass., 1954, 640 pp., $6

A compilation of latest editions of 33 fire safety standards in the field of building construction and equipment, covering such items as protection of openings in walls and partitions, garages, building exits, roofs, chimneys, industrial buildings, houses, construction operations in buildings, bridges and tunnels, fire tests, and others. The standards are not building codes, and are not intended for such use, although much of the material will be found useful in connection with the drafting and application of building codes and specifications.

Concrete products industry in Sweden (Betongvaruindustrin i Sverige)

H. Rohfors, *Betong* (Stockholm), V. 38, No. 4, 1953, pp. 269-274

Reviewed by Eivind Hognestad

The Swedish concrete products industry was founded in 1880. Decorative cast stone products dominated the first 20 years, later concrete pipe became the most important product. Extensive rationalization was started about 1940, and quality of the product has been improved. Engineering staffs of factories are being increased as a result of the use of expensive new equipment, and development of new products such as transmission poles, railroad ties, and large structural units.

Modern portland cement production

William Q. Hull, Peter Hass, and Paul Franklin, *Industrial and Engineering Chemistry*, V. 46, No. 5, May 1954, pp. 830-842

Brief history of portland cement production and uses. Definitions of portland cement are discussed. Manufacture of portland cement in a wet-process plant is described.

Conference on Swedish concrete research in progress (Konferens om pagaende svensk betongforskning)

Betong (Stockholm), V. 38, No. 4, 1953, pp. 205-246
Reviewed by EIVIND HOGNESTAD

A conference was held at Stockholm in November, 1953, to discuss and correlate research work at various Swedish laboratories and institutions. Brief reports on research projects in progress were followed by discussion. Nine groups of projects were reported:

1. *Properties of hardened concrete*—Theories of failure for concrete; influence of time and temperature on compressive strength; statistical variations of strengths of concrete and cement; test methods on permeability, compressive strength, shrinkage, modulus of elasticity, and frost resistance; strength and permeability of vibrated plain concrete pipe; uneven shrinkage resulting from segregation; properties of frozen concrete; heat conductivity in connection with radiant heating; residual stresses; comparison of lossier cement and portland cement in mortars; expansion force of lossier cement concrete.

2. *Static and dynamic behavior of concrete structures*—Characteristic functions of rectangular plates; effects of explosions and impact on concrete structures; dynamic bridge tests; model tests of flat slab floor; compression failures in continuous beams; temperature stresses in indeterminate structures.

3. *Prestressing problems*—Experimental comparison of beams reinforced with HJS 70 wires and beams reinforced by Freyssinet's system; studies of pre-tensioned prestressed concrete.

4. *Control problems*—Control of cement; control of water content in concrete; strength control of pavement concrete; control of precast concrete quality.

5. *Admixtures in concrete*—Influence on cement paste of surface active admixtures; admixtures in concrete; effect of air content on concrete strength.

6. *Reinforcement problems*—Standardization and testing of reinforcing steel; stress measurements on reinforcement; study of bond and anchorage; effects of direction of reinforcement with respect to principal stresses in plates.

7. *Cellular concrete*—New test method for heat conductivity; heat flow measurements; frost resistance of cellular concrete; effects of size and shape of specimen on compressive strength; exterior walls of cellular concrete.

8. *Concrete pavements*—Thin reinforced pavements; damage to the Air Force's pavements; concrete for pavements; scaling of highway pavements; experiences with concrete highways.

9. *Other problems*—Coarse aggregate content in trial mixes; determination of bleeding in concrete; suitability of various concrete aggregates; use of plastics for waterproofing mortars and concretes; uses of cement in mortars and plaster; placing concrete in long steel pipe piles; joints in concrete water tanks; wood-chip concrete; moisture in foundation walls; water flow by capillary suction in cellular concrete.

The conference terminated with a discussion of gaps in Swedish concrete research.

Effect of curing temperature on the compressive strength of soil-cement mixtures

K. E. CLARE and A. E. POLLARD, *Geotechnique* (London), V. 4, No. 3, Sept. 1954, pp. 97-107
Reviewed by ARON L. MURSKY

Five soils (a clay, a silty clay, two sands, a gravel) were studied. Compressive strength was found to increase with curing temperature; results indicate one mechanism of hardening is involved for cohesive soils up to 45 C, while more than one mechanism of hardening appears to be involved for the silty clay at 60 C and 100 C. Laboratory methods, especially the moisture-proof coatings used to prevent loss of moisture during curing, are discussed.

Protection of coastal lands against flooding and erosion

J. DUVIVIER, *Engineering* (London), V. 178, No. 4626, 4627; Sept. 24, 1954, pp. 404-407; Oct. 1, 1954, pp. 428-431
Reviewed by ARON L. MURSKY

Descriptions of various types of sea defenses in England, bearing out author's point that coast protection is less of an exact science than any other branch of civil engineering, due to the large number of variables involved, and that no other branch of engineering depends to such an extent on the experience and instinct of the designer.

Abridged version of paper presented before Section G (Engineering) of the British Assn. for the Advancement of Science, Oxford, Sept. 6, 1954. A summary of the discussion of this paper at the meeting is printed on p. 425 of the Oct. 1 issue of *Engineering*.

Exploration principles for major engineering works

W. R. Judd, *Proceedings*, ASCE, Separate No. 550, V. 80, Nov. 1954, 30 pp., $0.50

Author's Summary

Presents guiding principles for the formulation of exploration programs for various engineering structures. The power plant section discusses various design factors which affect the development of geologic investigations for this type of structure. Similarly, basic principles for developing exploration programs for canals, water tunnels, and earth and concrete dams are discussed. Power plant exploration is discussed in somewhat more detail than any of the other structures.

Soniscope applied to maintenance of concrete structures

Siegfried M. Breuning and A. J. Bone, *Proceedings*, Highway Research Board, V. 33, 1954, pp. 210-216

Authors' Summary

Testing techniques for various types of structures are discussed. Tests on prestressed concrete beams loaded to failure show a close correlation of Soniscope readings with the behavior of the beam. The influence of reinforcing steel is shown by tests on a reinforced bridge pier. The detection of cracks and deterioration in concrete are demonstrated by tests of several bridge members. A graphical presentation of Soniscope results on a bridge pier is then discussed. The results of these tests indicate that the Soniscope can be a valuable supplement to visual inspection.

Recent civil engineering works in Japan (in Japanese and English)

Japan Society of Civil Engineers, Tokyo, 1954, 252 pp.

Presents a pictorial record of the achievements and progress of the civil engineering profession in Japan. All photographs are excellent and many of the subjects are most impressive from an engineering point of view. A few of the major categories of structures shown are: soil conservation; power dams; water storage dams; shore protection structures; sanitary engineering structures; reconstruction of bomb damaged buildings; concrete highways; bridges of all sorts; other transportation forms including subways and an overhead monorail; modern heavy equipment in use in Japan, both native manufactured and imported; and a considerable section on model research. The application of precast concrete elements in particular is outstanding in the soil conservation and shore protection categories of structures including precast jetties, revetments, piles, tripods, articulated levee blanket covers, and blocks of various shapes. The book will be valuable to the civil engineer interested in an over-all view of professional progress. It is a unique summary showing examples of achievement in almost every phase of civil engineering. An American will be impressed by the close parallel to our own advances.

Striking examples selected at random are an elevated concrete water tank that looks like a temple; reinforced concrete buildings standing alone in demolished cities (1946) and in the middle of clean, modern cities (1954); bridges as familiar as those on American highways of all-welded plate girders, composite deck, and prestressed concrete beams; the heavy equipment at work, including all familiar U. S. brands among Japanese makes; and even an artificial earthquake generator in the model research section. This book definitely belongs in all engineering school libraries and will be an asset to many a private library.

Handyman's concrete and masonry guide

Robert Scharff, Arco Publishing Co., New York, N. Y., 1954, 144 pp., $2

Written for the layman, elementary instructions are given on mix proportioning, placing and handling of concrete, and brick and concrete masonry construction. Projects covered include sidewalks, driveways, porches and steps, terraces, retaining walls, septic tanks, swimming pool, concrete block garage, and miscellaneous concrete projects. Repairs and maintenance for walls, floors, and basements are also covered.

Reinforced concrete and prestressed concrete structures (Strutture de calcestruzzo armato e di calcestruzzo precompresso)

Riccardo Morandi, Libreria Dedalo Editric, Rome, Italy, 1954, 141 pp., 4600 lire

Magazine of Concrete Research
Sept. 1954

A record of the outstanding structures designed by Dr. Morandi. The book consists of a series of photographs, accompanied by drawings, and by text in Italian and English describing the buildings and bridges and the problems of design they raised and how these were solved.